Tuesday 13 June 2023 – Morning

Level 3 Cambridge Technical in Engineering

05823/05824/05825/05873 Unit 23: Applied mathematics for engineering

Time allowed: 2 hours
C305/2306

You must have:
- the Formula Booklet for Level 3 Cambridge Technical in Engineering (inside this document)
- a ruler (cm/mm)
- a scientific calculator

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) ____________________

Last name ____________________

Date of birth D D M M Y Y Y Y

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.

INFORMATION
- The total mark for this paper is **80**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **16** pages.

ADVICE
- Read each question carefully before you start your answer.

1 A pen is at a point B on a flatbed plotter. The point B is defined by the position vector $3\mathbf{i} + 2\mathbf{j}$ relative to an origin O, where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors. The pen is then moved through three sequential steps defined by direction vectors

$-4\mathbf{i} + \mathbf{j}$, $3\mathbf{i} - 3\mathbf{j}$,   and $2\mathbf{i} - \mathbf{j}$ respectively.

The pen ends at point E.

(i) Calculate the final location of the pen, point E, giving your answer as a position vector relative to the origin O.

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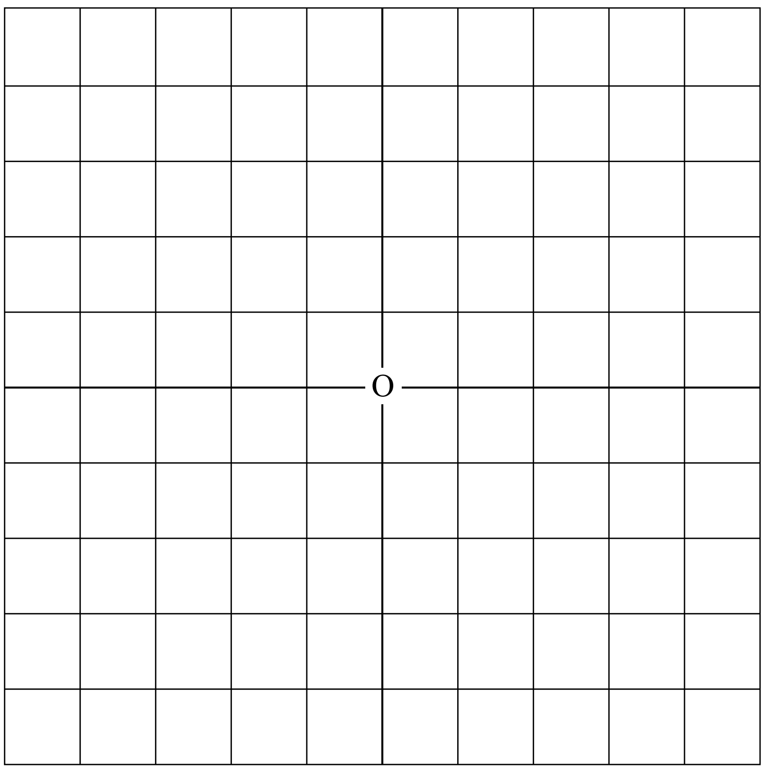
(ii) Mark points B and E on the grid below and draw labelled arrows to show the movements of the pen.



[3]

(iii) Calculate the direct distance between points B and E.

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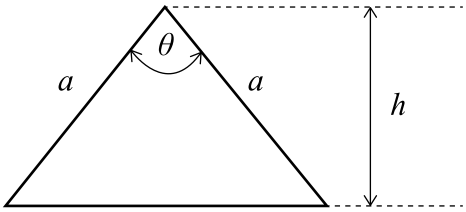
2 An isosceles triangle, shown below, has two equal sides of fixed length a. The height of the triangle, h, varies as the angle at the apex, θ, varies.



(i) Use calculus to show that the area of the triangle is maximised when

$h = \frac{a}{\sqrt{2}}$.

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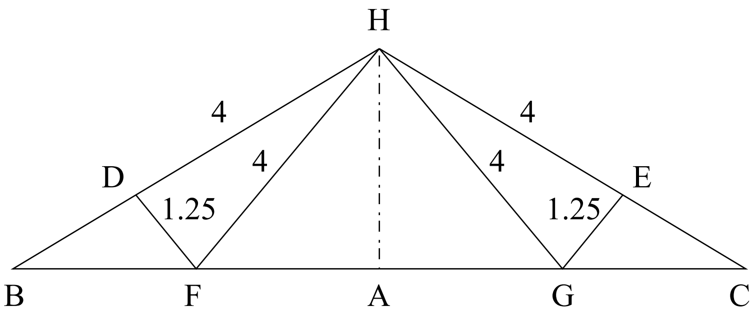
A diagram of a roof-supporting truss made of straight, rigid members is shown below.



Not to Scale

The length of members HE, HG and GE are 4 m, 4 m and 1.25 m respectively. The structure is symmetrical about the centre line, AH, and is arranged in such a way that the area of the triangle in the centre, FHG, is maximised.

(ii) State the distance AH.

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(iii) Calculate angle EHG giving your answer correct to the nearest degree.

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(iv) Calculate angle HGE giving your answer correct to the nearest degree.

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(v) Calculate the total width of the truss, BC.

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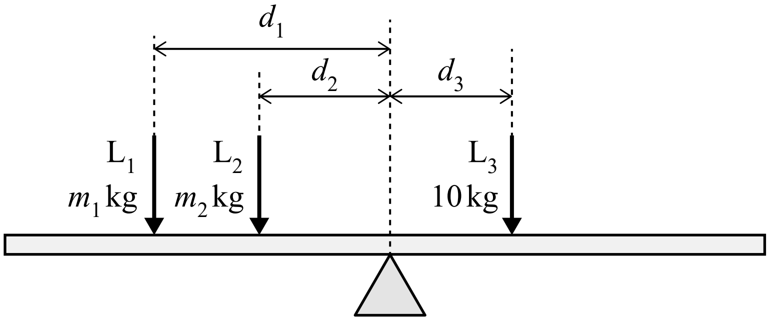
3 The lever, shown below, has loads L_1 and L_2 on one side of the fulcrum and a load L_3 on the other side of the fulcrum.



The distance between the fulcrum and L_1 is d_1 m, the distance between the fulcrum and L_2 is d_2 m and the distance between fulcrum and L_3 is d_3 m. Load L_1 has a mass of m_1 kg, load L_2 has a mass of m_2 kg and load L_3 has a mass of 10 kg. The lever has uniform mass which is centred at the fulcrum.

When $d_1 = 3$, $d_2 = 2$ and $d_3 = 1.45$ m the lever is in perfect balance.

When $d_1 = 1$, $d_2 = 5$ and $d_3 = 1.35$ m the lever is also in perfect balance.

(i) Use the information above to formulate two linear simultaneous equations in unknown masses m_1 and m_2.

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(ii) Express these equations in matrix notation.

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(iii) Use matrix methods to calculate the values of m_1 and m_2.

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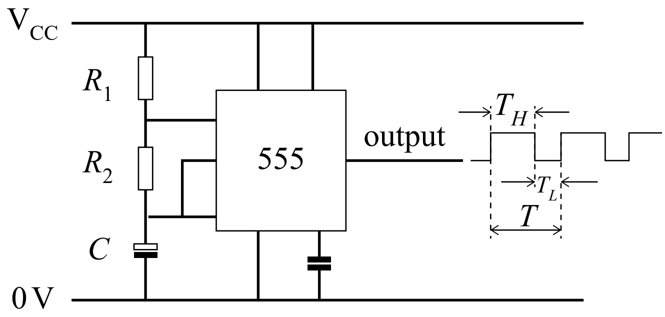
4 A '555 timer' integrated circuit can be configured to produce a stabilised square waveform oscillating with a frequency, f, of up to 500 kHz. Once configured the output voltage continually alternates between high and low states. The time, T_H s, that the output remains high and the time, T_L s, that the output remains low are controlled by two resistors, with values $R_1\,\Omega$ and $R_2\,\Omega$ and a capacitor with value C F. These are shown below within a simple 555 oscillator circuit.



For circuits of this type:

$T_H = \ln(2) \times (R_1 + R_2) \times C$ and $T_L = \ln(2) \times R_2 \times C$.

The period of oscillation, T s, is given by $T = T_H + T_L$.

The frequency of oscillation, f Hz, is given by $f = \frac{1}{T}$.

(i) A circuit of the type shown above is constructed with $R_1 = 1000$, $R_2 = 2000$ and $C = 10^{-5}$.

Calculate the frequency of oscillation, f.

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(ii) Another circuit of the type shown on page 8 is to be constructed so that it oscillates at a frequency of 0.5 Hz, with $C = 10^{-4}$ and $T_H = 1.2$.

Calculate the values of R_1 and R_2 correct to three significant figures.

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For this type of circuit the duty cycle, $DC\%$, is defined as:

$$DC = \frac{T_H}{T} \times 100,$$

where T_H and T are defined in terms of R_1, R_2 and C as shown immediately below the circuit on page 8.

(iii) Show that $DC = \frac{(R_1 + R_2)}{R_1 + 2R_2} \times 100$.

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(iv) Assuming that $R_1 + R_2 > 0$ use the result in part **(iii)** to determine the theoretical maximum and minimum values of DC.

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5 For this question use the relationship $a = \frac{dv}{dt}$,

where a is acceleration,
v is speed,
t is time.

A passenger lift in a tall building starts from rest at the top floor and descends without stopping to the ground floor in 50 seconds. During its descent the acceleration of the lift, $a\,\text{m}\,\text{s}^{-2}$, is modelled by the equation

$a = 0.003t^2 - 0.2t + 2.5$,

where t s is the time from the start of its descent.

(i) Calculate the speed of the lift, $v\,\text{m}\,\text{s}^{-1}$, when $t = 10$.

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(ii) Calculate the maximum speed reached by the lift during its descent.

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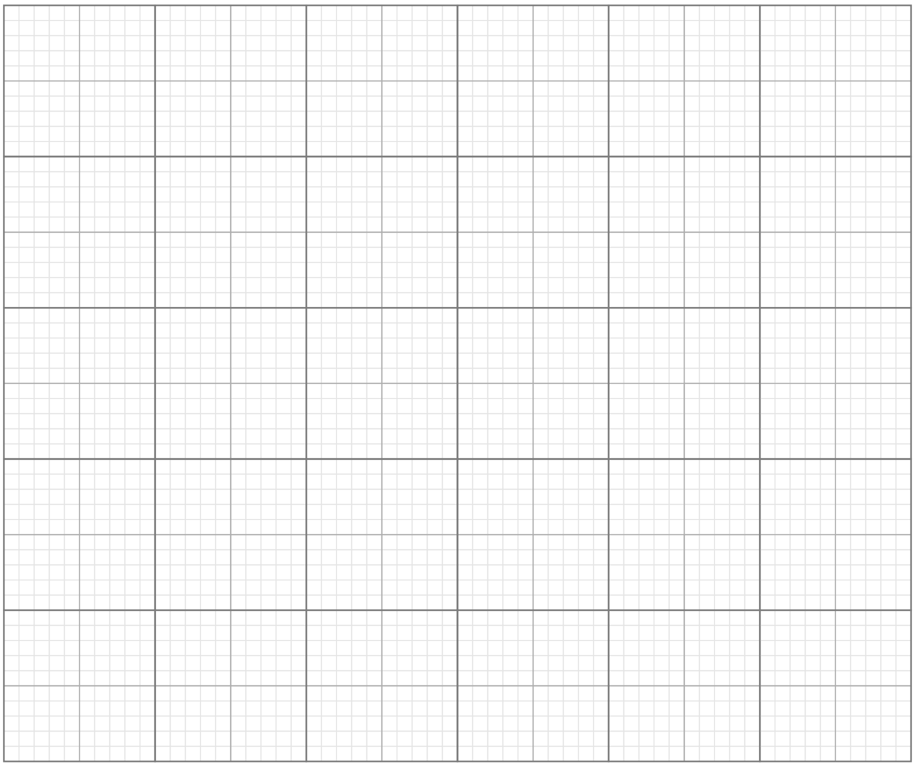
(iii) Draw a graph of v against t for $(0 \leqslant t \leqslant 50)$ on the grid below.



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(iv) For this part of the question you are given the following information.

The distance s travelled by the lift in a period from t_0 to t_1 is given by

$$s = \int_{t_0}^{t_1} v \, \mathrm{d}t.$$

Using this information calculate the total distance travelled by the lift during its descent.

[4]

6 For a communications system the impulse response, h(t), and its frequency response, H(f), are related by the integral

$$H(f) = \int_{-\infty}^{\infty} h(t)\, e^{-j2\pi ft}\, dt,$$

where $j = \sqrt{-1}$, t s is time and f Hz is frequency.

For a particular communications system h(t) is defined as

$h(t) = e^{-t}$ for $t \geqslant 0$,

$h(t) = 0$ for $t < 0$.

(i) Show that $H(f) = \dfrac{1}{1 + j2\pi f}$

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Now consider the case when $f = 0.5$.

(ii) Express H(0.5) in the form $a + jb$ where a and b are real values correct to three decimal places.

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(iii) Plot H(0.5) on an Argand diagram.

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(iv) Express H(0.5) in the form $r(\cos\theta + \mathrm{j}\sin\theta)$ giving values for r and θ.

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7 The variation in sea level each day affects the amount of electrical energy that can be generated using tidal energy. The tidal range is defined as the difference between high sea water level and low sea water level, and this varies due to the gravitational effect of the Moon. You should assume that the Moon's orbit round the Earth is circular, and that the Moon moves through 2π radians in a lunar month of 30 days. The tidal range, h m, at a particular place is modelled by the formula

$h = 4(\cos^2 \theta + 1)$

where θ is the angle through which the Moon has moved around the Earth from the start of a lunar month.

(i) Calculate, in radians, the angle the Moon moves through in the first 5 days of a lunar month.

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(ii) The average value, $\overline{y}$, of a function $y = \mathrm{f}(x)$ over the interval $a \leqslant x \leqslant b$ is given by

$$\overline{y} = \frac{1}{b-a}\int_a^b \mathrm{f}(x)\,\mathrm{d}x.$$

Calculate the average tidal range in the first 5 days of the lunar month.

You may use $\cos^2 A = \frac{1}{2}(\cos(2A) + 1)$.

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A tidal lagoon is an area of sea enclosed by a wall. The wall contains underwater turbines that generate electrical energy when water passes through them as the tide rises and falls. Water passes through the turbines four times each day, and each time the electrical energy generated, E MW h, is approximated by the formula

$$E = \frac{1}{2} A \rho g h^2 \times \frac{10^{-6}}{3600},$$

where $A\,\text{m}^2$ is the surface area of lagoon,
$\rho\,\text{kg}\,\text{m}^{-3}$ is the density of sea water,
$g\,\text{m}\,\text{s}^{-2}$ is acceleration due to gravity,
$h\,\text{m}$ is the tidal range.

You are given that $\rho = 1025$.

The surface area of a proposed lagoon is $11.5 \times 10^6\,\text{m}^2$.

On a particular day the tidal range is 5.3 m.

(iii) Estimate the total electrical energy that would be generated on that day.

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(iv) Give reasons why the actual energy generated may be significantly different from the energy calculated in part **(iii)**.

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END OF QUESTION PAPER