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AS LEVEL

Examiners' report

MATHEMATICS A

H230

For first teaching in 2017

H230/02 Summer 2023 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from our secure Teach Cambridge site (https://teachcambridge.org).

Paper 2 series overview

H230/02 is one of the two examination components for AS Mathematics A. The examination is structured in two sections; Section A: Pure Mathematics (this particularly question paper consisted of eight questions allocated 50 marks) and Section B: Mechanics (which consisted of three questions allocated 25 marks). All questions should be answered. Each section has a gradient of difficulty throughout the section and consists of a mix of short and long questions.

Three overarching themes are applied across all content.

1. Mathematical argument, language and proof.

2. Mathematical problem solving.

3. Mathematical modelling.

To do well on this paper candidates need to be comfortable applying their knowledge and understanding to all three of these overarching themes, in both familiar and unfamiliar contexts.

Teachers and candidates are encouraged to study carefully the requirements we expect from the command words such as 'Determine', 'Show that', 'Hence', **'In this question you must show detailed reasoning' (DR)**, etc. These are explained in the specification document, as well as being covered in our command words poster available from the 'For your students' section of Teach Cambridge. This year there was some evidence of improved understanding of command words by candidates and it would be good to reinforce their meaning at every opportunity in the classroom with future students, to see this improvement continue. The below comment on specific questions highlights common candidate errors and misconceptions leading to loss of marks, as well as good practice that was seen.

We expect, and encourage, candidates to use their calculator effectively. In **DR** questions however, it must again be stressed that we want to **see** methods used by candidates in their response; calculator use must be consistent with this requirement (see the comments on Questions 2 and 8 below).

There was no evidence that time constraints led to a candidate underperforming.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• understood well the concepts tested in both sections of the paper • were accurate, both numerically and algebraically • produced neat, well-argued solutions • gave responses as requested (see in particular Questions 3 (c), 4 (b), 6 (d) and 7 (a)) • coped well with modelling questions (see Questions 3 and 11) and extended questions.	• had a more limited understanding of the concepts in both sections of the paper • were prone to both numerical and algebraic errors (see the comments below for Question 8) • were untidy, perhaps with insufficient working • did not give responses as requested (see Questions 6 (d) and 7 (a) especially) • struggled to identify the required mathematical methods in modelling situations • found extended questions challenging.

Section A overview

This is a 1 hour 30 minute paper. Section A comprised 50 out of the paper's 75 marks, which candidates are strongly recommended to consider when deciding how long to spend on this section.

The front cover rubric states 'Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question' and this is worthy of note in the context of Questions 4(a) and 8 here.

Question 1

1 The quadratic equation $kx^2 + 3x + k = 0$ has no real roots.

Determine the set of possible values of k. **[3]**

Most candidates realised that the discriminant of the quadratic equation needed to be considered. Many knew that $\Delta < 0$ was needed, although the first 2 marks were potentially available from $\Delta = 0$ or $\Delta > 0$. Critical values of $\pm\frac{2}{3}$ and $\pm\frac{3}{4}$ were seen and it was not uncommon to only see a positive critical value. The final response was sometimes given as $k > \pm\frac{3}{2}$, or $-\frac{3}{2} < k < \frac{3}{2}$ and we did not give A1 if 'and' was used in the final statement, regarding this as equivalent to the set notation $\cap$.

Question 2

2 **In this question you must show detailed reasoning.**

Solve the equation $x\sqrt{5} + 32 = x\sqrt{45} + 2x$. Give your answer in the form $a\sqrt{5} + b$, where a and b are integers to be determined. **[4]**

Many candidates gained the B1 mark. Some solutions then floundered because the need to factorise out x was not understood. Those who did this however usually reached the correct answer, but it was important to show how this was achieved (note the comments below); the rationalising process needed to be clear and we expected to see some working before the final answer.

The alternative scheme was rarely seen.

Assessment for learning

This question starts with the instruction '**In this question you must show detailed reasoning**'. This is an instruction to candidates to show sufficient evidence and detail in their solution. Calculators may be used, but the response candidates provide should be clear to follow and set out the working showing each stage. For example in this question, providing working showing the method for rationalising the surd. This comment is consistent with that made in the 2022 report on Question 2.

Exemplar 1

$x\sqrt{5} + 32 = x\sqrt{45} + 2x$

$x\sqrt{5} + 32 = 3x\sqrt{5} + 2x$

$2x\sqrt{5} + 2x - 32 = 0$

$x\sqrt{5} + x - 16 = 0$

~~assume $x =$ ...~~

$x(1+\sqrt{5}) - 16 = 0$

$x(1+\sqrt{5}) = 16$

$x = \frac{16}{1+\sqrt{5}}$

$x = -4 + 4\sqrt{5}$

$x = 4\sqrt{5} - 4$

$a = 4 \quad b = -4$

This exemplar illustrates the point raised above. The candidate does not indicate how they have reached 'x = -4 + 4√5' from the preceding line, which is not sufficient for the M1dep* to be given.

Question 3 (a) (i)

3 A Ferris wheel at a fairground rotates in a vertical plane. The height above the ground of a seat on the wheel is h metres at time t seconds after the seat is at its lowest point.

The height is given by the equation $h = 15 - 14\cos(kt)°$, where k is a positive constant.

(a) **(i)** Write down the greatest height of a seat above the ground. **[1]**

An incorrect response of 15 m was not uncommon.

Question 3 (a) (ii)

(ii) Write down the least height of a seat above the ground. **[1]**

Most candidates answered this correctly, with 0 m being the most common alternative.

Question 3 (b)

(b) Given that a seat first returns to its lowest point after 150 seconds, calculate the value of k. **[2]**

It was not unusual to see cos(150*k*) = 1 lead to 150*k* = 0 and confusion. Elsewhere, cos(150*k*) = 0 appeared too, from $\frac{14}{14}$ = 0 and occasionally cos(150*k*) = -1 from sign errors. Those who had 0 m as their answer in (a)(ii) got cos(150*k*) = $\frac{15}{14}$ and often stopped there. Another error was candidates evaluating cos(150*k*) as 150cos*k* or *k*cos150.

Question 3 (c)

(c) Determine for how long a seat is 20 metres or more above the ground during one complete revolution. Give your answer correct to the nearest tenth of a second. **[4]**

A correct response here required candidates' value for *k* from (b), but those who were unsuccessful in (b) usually did not attempt this part. Candidates who had found a value for *k* were often able to pick up the first 2 marks here, but many found just one value for *t* and gave this as the final answer. Sign errors sometimes led to cos(*kt*) = $\frac{5}{14}$.

Question 4 (a)

4 **(a)** Find and simplify the first three terms in the expansion, in ascending powers of x, of $\left(2+\frac{1}{3}kx\right)^6$, where k is a constant. **[3]**

This question was answered well. The binomial series is included on the formulae sheet and this was no doubt of benefit to candidates, with many having at least two correct terms. The question said, 'Find and simplify…', so examiners expected to see the terms simplified and answers such as $240/9k^2x^2$ scored A0 (this term was the one that candidates most often struggled to simplify, with $80k^2x^2$, $80kx^2$ and $80/3kx^2$ seen). Occasionally the three terms were given in descending powers of *x*.

Question 4 (b)

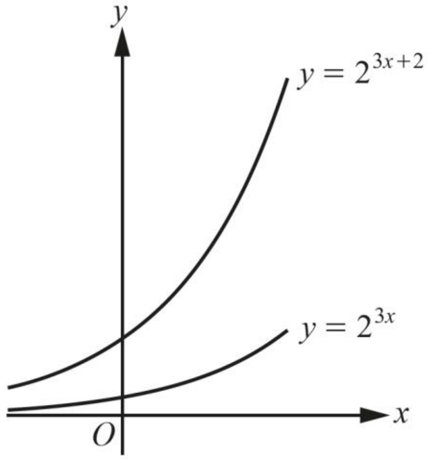
(b) In the expansion of $(3-4x)\left(2+\frac{1}{3}kx\right)^6$, the constant term is equal to the coefficient of x^2.

Determine the exact value of k, given that k is positive. **[3]**

This part of the question proved more challenging. Many candidates obtained the two term coefficient of x^2 in the correct way, but this was then set equal to *k* or 0 rather than 3 × their constant term from (a). Note that those having *k* rather than k^2 in the x^2 term in (a) could not get a three term quadratic in *k*. The request for an exact value for *k* was well adhered to.

Question 5 (a)

5



The diagram shows the graphs of $y = 2^{3x}$ and $y = 2^{3x+2}$. The graph of $y = 2^{3x}$ can be transformed to the graph of $y = 2^{3x+2}$ by means of a stretch.

(a) Give details of the stretch. **[2]**

The mark scheme gives comprehensive detail about the wording that was accepted and this is well worth studying carefully. Giving the scale factor as 2 or $\frac{1}{2}$ was fairly common, as was 'in the *y*-axis'.

Question 5 (b)

The point A lies on $y = 2^{3x}$ and the point B lies on $y = 2^{3x+2}$. The line segment AB is parallel to the y-axis and the difference between the y-coordinates of A and B is 36.

(b) Determine the x-coordinate of A. Give your answer in the form $m\log_2 n$ where m and n are constants to be determined. **[3]**

Most attempts gained 1 mark, for finding the correct equation in x. Using $2^{3x+2} = 4(2^{3x})$ to obtain the equation $2^{3x} = 12$ was very rare. The exemplar below shows the typical next step in many solutions. Less successful efforts did not involve 36 at all.

Exemplar 2

Difference between y coordinates is 36

$2^{3x+2} - 2^{3x} = 36$

~~2^{3x+2}~~ $3x + 2\log(2) - 3x\log(2) = \log(36)$

Taking logs like this was very prevalent and often led to the solution being abandoned quickly (presumably because *x* disappears). Incorrect use of the logarithm laws was frequent in this question.

Question 6 (a)

6 The vertices of triangle ABC are $A(-3,1)$, $B(5,0)$ and $C(9,7)$.

(a) Show that $AB = BC$. **[2]**

This was done well by most candidates. A few lost the answer mark through careless labelling, e.g. $AB = 65$. Exact values were expected, so working with decimals only also lost the answer mark.

Question 6 (b)

(b) Show that angle ABC is **not** a right angle. **[2]**

This was also well done, but as a 'Show that…' question we did expect candidates' calculations to be involved in the conclusion, for whichever method they chose.

Question 6 (c)

(c) Find the coordinates of the midpoint of AC. **[1]**

There were many correct responses here. The most common incorrect response was (6, 3).

Question 6 (d)

(d) Determine the equation of the line of symmetry of the triangle, giving your answer in the form $px + qy = r$, where p, q and r are integers to be determined. **[2]**

Those candidates who understood that the line *BM* was expected often gained full marks, although some candidates did not give their response in the requested form. Some thought the line *AC* was needed. Most attempts started by finding a gradient and then using it with one point in either $y = mx + c$ or $y - y_1 = m(x - x_1)$. Using $m = \Delta x/\Delta y$ was given M0.

Question 6 (e)

(e) Write down an equation of the circle with centre A which passes through B. **[2]**

The form of the equation of a circle seemed well known. A subtraction sign was seen on the left-hand side in a few cases, as well as *AB* rather than AB^2 on the right-hand side.

Question 6 (f)

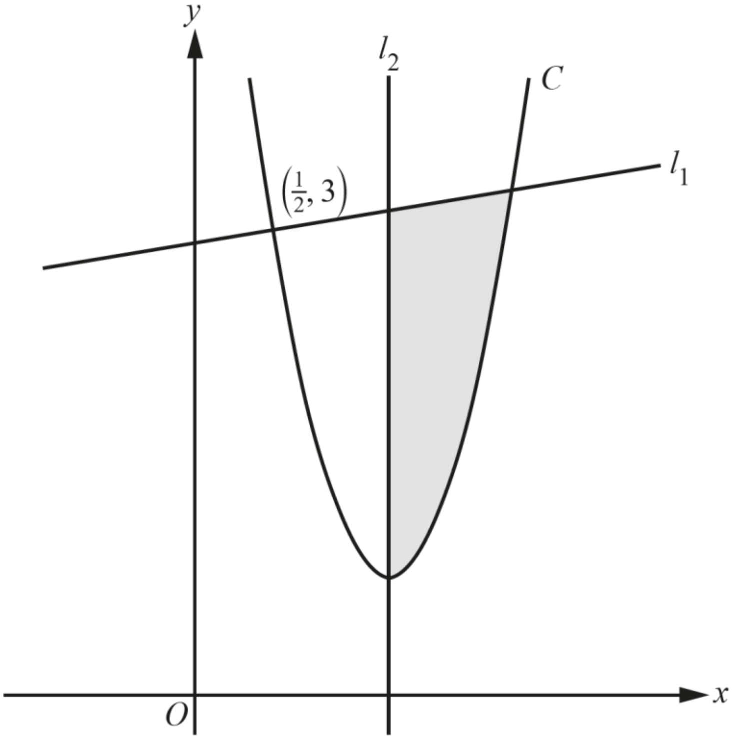
This circle intersects the line of symmetry of the triangle at B and at a second point.

(f) Find the coordinates of this second point. **[1]**

Most responses that earned this mark solved their correct equations from (d) and (e) simultaneously, producing rather a lot of working. Very few candidates used the more efficient method involving similar triangles.

Question 7 (a)

7



The diagram shows the curve C with equation $y = 4x^2 - 10x + 7$ and two straight lines, l_1 and l_2. The line l_1 is the normal to C at the point $\left(\frac{1}{2}, 3\right)$. The line l_2 is the normal to C at the minimum point of C.

(a) Determine the equation of l_1, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be determined. **[4]**

The majority of candidates obtained the correct derivative (i.e. $8x - 10$) and those who used it correctly with the given point mainly scored full marks. As was the case elsewhere, a few lost the final mark by not giving their answer in the requested form (e.g. $x - 6y + 17.5 = 0$, from candidates either not spotting or misunderstanding the request for integers). A significant number of responses took the gradient of l_1 to be 8 straight from the derivative, or $\frac{10}{8}$ (from setting their derivative to 0) and then finding the negative reciprocal. Using $y = mx + c$ to find the normal was popular.

Question 7 (b)

The shaded region shown in the diagram is bounded by C, l_1 and l_2.

(b) Determine the inequalities that define the shaded region, including its boundaries. **[3]**

This part proved challenging. $x \geq 1.25$ appeared most often, sometimes as the only inequality given. Candidates felt more comfortable obtaining an inequality from their normal when it had been written 'y =….'. Some used strict inequalities. Less successful responses had inequalities involving the labels l_1, l_2 and C.

Question 8

8 In this question you must show detailed reasoning.

Given that $\int_4^a \left(\frac{4}{\sqrt{x}}+3\right)\mathrm{d}x = 7$, find the value of a. **[7]**

The majority of responses gained the first method mark, but there was some careless work with powers when dealing with $4x^{-½}$. Some candidates used $4x^{-1}$ or $4x^{½}$, which was costly.

After integration, $2x^{½} + 3x$ spoiled some efforts. A few embarked on the question without integrating at all. The use of limits was generally good, with only the occasional candidate having the wrong order or adding terms. The majority of candidates that reached the 'hidden' quadratic did identify it, but as this is a **DR** question, we did want to see how this quadratic was solved; those who did it by calculator without showing working lost the last method mark (the 'Assessment for learning' comment above for Question 2 is relevant again here). We expected the final answer to be exact.

Section B overview

This is a 1 hour 30 minute paper. Section B comprised 25 out of 75 marks and as with Section A, candidates are strongly recommended to consider this when deciding how long to spend on this section.

While most questions do not require the units to be stated in the final answer, it is sensible for candidates to get into the habit of always providing them.

Question 9 (a)

9 A cyclist travels along a straight horizontal road between house A and house B.

The cyclist starts from rest at A and moves with constant acceleration for 20 seconds, reaching a velocity of $15\,\mathrm{m\,s^{-1}}$. The cyclist then moves at this constant velocity before decelerating at $0.3\,\mathrm{m\,s^{-2}}$, coming to rest at B.

(a) Find the time, in seconds, for which the cyclist is decelerating. **[1]**

The vast majority of responses were correct.

Question 9 (b)

(b) Sketch a velocity-time graph for the motion of the cyclist between A and B. [Your sketch need not be drawn to scale; numerical values need not be shown.] **[1]**

Most sketches were of a trapezium, but we did expect the gradients of the lines representing the first 20 seconds and last 50 seconds to reflect the given information.

Question 9 (c)

The total distance between A and B is 1950 m.

(c) Find the time, in seconds, for which the cyclist is moving at constant velocity. **[2]**

There were quite a lot of correct responses presented, with many showing 1950 – 150 – 375 = 1425 and then $\frac{1425}{15}$ = 95s (i.e., not starting with an equation in T). Some candidates thought the first and last distances were 15 × 20 and 15 × 50, while others thought $T = \frac{1950}{15}$. Those who worked with a time that wasn't that specifically requested (e.g. the time up to the end of the constant velocity) did not always conclude by giving what we requested.

Question 10 (a)

10 A particle P is moving in a straight line. At time t seconds, where $t \geqslant 0$, P has velocity $v\,\mathrm{m\,s^{-1}}$ and acceleration $a\,\mathrm{m\,s^{-2}}$ where $a = 4t - 9$. It is given that $v = 2$ when $t = 1$.

(a) Find an expression for v in terms of t. **[3]**

This question was answered very well, with the integration generally correct. A few candidates omitted '+ *c*'. Using $v = (4t - 9)t$ was given M0.

Question 10 (b)

The particle P is instantaneously at rest when $t = t_1$ and $t = t_2$, where $t_1 < t_2$.

(b) Find the values of t_1 and t_2. **[2]**

Those who were successful with (a) usually scored full marks here.

Question 10 (c)

(c) Determine the total distance travelled by P between times $t = 0$ and $t = t_2$. **[3]**

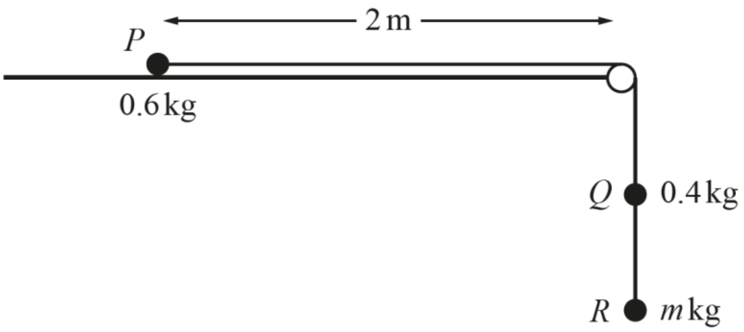
Most candidates realised this part required *v* to be integrated, with many not using the calculator to do this. The most common error was to just do this with limits 3 and 0 (mirroring a similar error seen in Question 11(c) of the summer 2022 paper); this scored no marks. With this year's question now giving future candidates an extra question of this type to practice on in mock exams, hopefully this type of error will be reduced in the future.

Assessment for learning

When doing definite integrals on the calculator, it is wise (as long as **time permits**) for candidates to type in the relevant details twice to hopefully avoid careless errors.

Question 11 (a)

11 Two balls P and Q have masses 0.6 kg and 0.4 kg respectively. The balls are attached to the ends of a string. The string passes over a pulley which is fixed at the edge of a rough horizontal surface. Ball P is held at rest on the surface 2 m from the pulley. Ball Q hangs vertically below the pulley. Ball Q is attached to a third ball R of mass m kg by another string and R hangs vertically below Q (see diagram).



The system is released from rest with the strings taut. Ball P moves towards the pulley with acceleration $3.5\,\text{m s}^{-2}$ and a constant frictional force of magnitude 4.5 N opposes the motion of P.

The balls are modelled as particles, the pulley is modelled as being small and smooth, and the strings are modelled as being light and inextensible.

(a) By considering the motion of P, find the tension in the string connecting P and Q. **[2]**

This part of Question 11 was done reasonably well. The question stated 'By considering the motion of *P*' to hopefully help focus candidates' minds, although some equations involved 0.6 g. Some less successful responses also introduced 0.4 into their equation(s).

Assessment for learning

Candidates should note what is acceptable when writing equations using Newton's Second Law. For the method mark here, the mark scheme requires 'correct number of terms and dimensionally consistent – allow sign confusion'.

Question 11 (b)

(b) Hence determine the value of m. Give your answer correct to **3** significant figures. **[4]**

We saw some good solutions to this part, but quite a lot of equations had extra or missing terms. In the equation for R the value found for T_1 was sometimes placed where T_2 should have been, resulting in this equation alone being used to find m. Those considering Q and R together here needed to be careful (see the comment in (a)). 04*m* was used in some equations. The request for 3 significant figures was well followed here.

Question 11 (c)

When the balls have been in motion for 0.4 seconds the string connecting Q and R breaks.

(c) Show that, according to the model, P does not reach the pulley. **[6]**

Many candidates earned the first 2 B marks for the motion until the string breaks, but what happened after this was often confused. Some just thought P only moved 0.28 m and compared this with 2 or used a = g in the subsequent motion. Others who attempted one or two Newton II equations used T = 0 or maybe 6.6. Use of the correct deceleration was uncommon.

Exemplar 3

$s_1 = ut + \frac{1}{2}at^2$ $u=0$ initially

$s_1 = \frac{1}{2}(3.5)(0.4)^2$

$= 0.28m$

$v = u + at$

$= (3.5)(0.4)$

$= 1.4ms^{-1}$

$s_2 = ut + \frac{1}{2}at^2$ $u = v$ $(1.4ms^{-1})$

$F = ma$

$4.5 = 0.6a$

$a = -7.5ms^{-2}$

~~s = 1.4~~

$v = u + at$

$0 = 1.4 + -7.5t$

$t = \frac{14}{75}$

$s_2 = (1.4)(\frac{14}{75}) + (\frac{1}{2})(-7.5)(\frac{14}{75})^2$

$= 0.131m$ (3sf)

$s_1 + s_2 = s_T = 0.28 + 0.131$

$= 0.414m$ (3sf) $<$ 2metres

The total distance travelled by P is less than 2 metres and therefore P does not reach the pulley.

This shows a candidate who realises that there is a change of acceleration but uses Newton II with $T = 0$.

Question 11 (d)

It is given that in fact ball P does reach the pulley.

(d) Identify one factor in the modelling that could account for this difference. **[1]**

The mark scheme provides examples of the sort of things we accepted here and it would be recommended for centres to review this with future students. Candidates should be encouraged to write their response in a full sentence, as it was not always clear what a candidate meant. For example, some candidates just wrote 'Friction', which could mean the frictional force opposing the motion of P is not constant (which means the ball could reach the pulley), or there is a frictional force on the pulley (which would make the ball less likely to reach the pulley). Some comments were difficult to read; we encourage candidates to always write their responses clearly. It is also worth noting that **one** factor was requested; if more than one factor was given then all had to be acceptable to gain the mark.

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