Qualification Accredited

OCR
Oxford Cambridge and RSA

A LEVEL

Examiners' report

MATHEMATICS A

H240

For first teaching in 2017

H240/03 Summer 2023 series

Version 1

ocr.org.uk/maths

Contents

Introduction 3
Paper 3 series overview 4
Section A overview 6
Question 1 6
Question 2 (a) 6
Question 2 (b) 7
Question 3 (a) (i) 7
Question 3 (a) (ii) 7
Question 3 (b) 8
Question 4 (a) 8
Question 4 (b) 8
Question 5 (a) 9
Question 5 (b) 10
Question 6 (a) 10
Question 6 (b) 11
Question 7 (a) 11
Question 7 (b) 12
Question 7 (c) 13
Question 7 (d) 13
Section B overview 14
Question 8 14
Question 9 (a) 14
Question 9 (b) 15
Question 10 (a) 16
Question 10 (b) 17
Question 11 (a) 17
Question 11 (b) 18
Question 12 (a) 18
Question 12 (b) 19
Question 12 (c) 19
Question 12 (d) 20
Question 13 (a) 21
Question 13 (b) 23

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Paper 3 series overview

This is the third and final examination component for the A Level examination for GCE Mathematics A. It is a two-hour paper consisting of 100 marks which tests content from Pure Mathematics (Section A, 50 marks) and Mechanics (Section B, 50 marks). Pure Mathematics content is tested on all three papers, and any topic could be tested on any of the three papers. Inevitably, the report that follows will concentrate on aspects of the candidates' performance where improvement is possible to assist centres on preparing candidates for future series. However, this should not obscure the fact that a significant number of candidates who sat this paper in this reformed A Level qualification produced solutions which were a pleasure for examiners to assess. Many candidates demonstrated a most impressive level of mathematical ability and insight which enabled them to meet the various challenges posed by this paper on both the pure and mechanics content; precision, command of correct mathematical notation and excellent presentational skills were evident in many scripts.

The specification includes some guidance about the level of written evidence required in assessment questions; these were provided to reflect the increased functionality of the available calculators and the changes in assessment objectives, since there is a significant change from when the equivalent legacy qualifications were designed.

There was one question on this paper which began with the demand '**In this question you must show detailed reasoning**'; to quote the specification that 'when a question includes this instruction candidates must give a solution which leads to a conclusion showing a detailed and complete analytical method. Their solution should contain enough detail to allow the line of their argument to be followed. This is not a restriction on a candidate's use of a calculator when tackling a question…but it is a restriction on what will be accepted as evidence of a complete method.' The specification then considers several examples which centres should consider so that future candidates understand exactly what is required when this request appears in future series. This command phrase features in Question 7 (a).

The word 'Determine' in a question does not simply imply that candidates should find the answer but, to quote the specification, 'this command word indicates that justification should be given for any results found, including working where appropriate.' This command word features in Questions 3 (b), 4 (b), 5 (b), 6 (a), 6 (b), 8, 9 (b), 10 (b), 11 (b), 12 (b), 12 (d), 13 (a) and 13 (b).

The phrase 'Show that' generally indicates that the answer has been given, and that candidates should provide an explanation that has enough detail to cover every step of their working. This command phrase features in Questions 5 (a), 7 (b), 10 (a), 11 (a).

While there is no specific level of working needed to justify answers to questions which use the command word 'Find …', method marks may still be available for valid attempts that do not result in a correct answer, and standard advice (included in the specification) that candidates should state explicitly any expressions, integrals, parameters and variables that they use a calculator to evaluate (using correct mathematical notation rather than model specific calculator notation).

OCR support

Full details of 'command words' can be found in section 2d of the specification. OCR also publish a classroom poster and associated guidance on command words, which centres should make sure candidates are familiar with. These can be found on Teach Cambridge.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• made efficient use of calculators • understood the level of response required for command words used in the questions • read questions carefully and provided the answers that were requested • used formal mathematical notation and language correctly.	• made careless mistakes in algebraic manipulation • used imprecise notation or language • provided mathematical working that was correct but did not answer the specific question that was being asked • did not give sufficient evidence in 'Show that' and 'Determine' questions.

Section A overview

Content from the Pure section of the specification may be assessed on any of the three papers of H240. Most candidates appeared well prepared for the pure content, with method marks being given, but there were a number of places where a more concise use of algebraic notation and language may have led to more of the corresponding accuracy marks being available.

Question 1

1 Using logarithms, solve the equation

$$4^{2x+1} = 5^x,$$

giving your answer correct to **3** significant figures. **[3]**

Examiners noted that candidates' responses to this question were mixed with many struggling with what would appear to be a routine demand of using logs to solve an equation of the form $a^{\mathrm{f}(x)} = b^{\mathrm{g}(x)}$. The most common error was to take logs of both sides with two different bases, e.g. $(2x+1)\log_5 4 = x\log_4 5$. For those candidates that did take logs correctly many then appeared to struggle to solve the resulting linear equation. A number of candidates left their answer in an exact form, e.g. $x = \frac{\log 4}{\log 5 - 2\log 4}$ and did not give the answer to 3 significant figures as requested.

Question 2 (a)

2 (a) Express $3\sin x - 4\cos x$ in the form $R\sin(x-\alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of α correct to **4** significant figures. **[3]**

This question was answered particularly well with almost all candidates stating the correct value for *R*. The most common errors when it came to determining the value of α was to suggest that

$\cos\alpha = 3$, $\sin\alpha = 4$ or to incorrectly state that $R\sin\alpha = -4 \Longrightarrow \alpha = -53.13$

$\therefore 3\sin x - 4\cos x = 5\sin(x - 53.13)$, so obtaining the correct answer from incorrect working (which was penalised with the final mark in this, and the next part). Furthermore, many candidates only gave the value of α to 3 significant figures (or in some cases as an exact value in terms of arctan).

Question 2 (b)

(b) Hence solve the equation $3\sin x - 4\cos x = 2$ for $0° < x < 90°$, giving your answer correct to **3** significant figures. **[2]**

Most candidates used the harmonic form from part (a) to obtain the correct value of x in this part. Occasionally candidates spent time finding further solutions that were not in the required range. Those candidates who ignored the advice of 'hence' and attempted to solve this equation by other methods where not penalised for not following the question request in this instance, however they were rarely completed the solution successfully.

'Hence' or 'Hence or otherwise'

Candidates should recognise that where a question uses the word 'hence' they are best advised to start their solution based on the previous work. If 'hence or otherwise' is used then alternative (perhaps using Further Maths techniques) routes may be appropriate, although these may be more time consuming or complex.

Question 3 (a) (i)

3 The cubic polynomial $f(x)$ is defined by $f(x) = x^3 + px + q$, where p and q are constants.

(a) (i) Given that $f'(2) = 13$, find the value of p. **[2]**

This part was answered extremely well with almost all candidates obtaining the correct answer of $p = 1$. The most common (rare) error in this part was a slip when differentiating x^3.

Question 3 (a) (ii)

(ii) Given also that $(x-2)$ is a factor of $f(x)$, find the value of q. **[2]**

Similarly, to part (a) (i) this part was answered extremely well. However, many candidates seemed to ignore the most straightforward method of applying the factor theorem directly as $2^3 + 2p + q = 0$ (and solving for *q* using their value of *p* from (i)) and instead used methods of factorisation or long division (with mixed success).

Question 3 (b)

The curve $y = f(x)$ is translated by the vector $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

(b) Using the values from part **(a)**, determine the equation of the curve after it has been translated. Give your answer in the form $y = x^3 + ax^2 + bx + c$, where a, b and c are integers to be found. **[4]**

Although most candidates realised that the given translation meant that the new curve would have the equation $y = (x-2)^3 + p(x-2) + q - 3$ some candidates incorrectly added the 3, or considered the horizontal translation as $(x+2)$. Some then struggled to expand $(x-2)^3$ correctly or did not give their final answer as an equation of '$y = \cdots$' (and simply left their answer as $x^3 - 6x^2 + 13x - 23$).

Question 4 (a)

4 A circle C has equation $x^2 + y^2 - 6x + 10y + k = 0$.

(a) Find the set of possible values of k. **[2]**

Although most candidates scored the first mark in this part for re-writing the equation as

$(x-3)^2 - 9 + (y+5)^2 - 25 + k = 0$ not all then realised that $(x-3)^2 + (y+5)^2 = 34 - k$ and therefore, for the equation to represent a circle, the right-hand side of this equation had to be positive (and hence $k < 34$).

Question 4 (b)

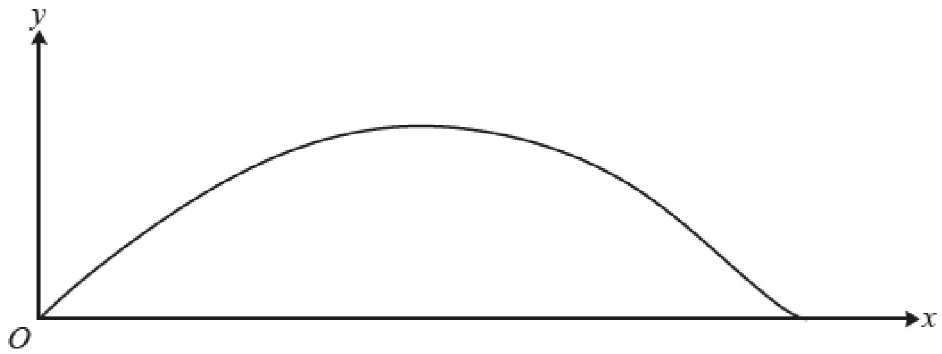
(b) It is given that $k = -46$.

Determine the coordinates of the **two** points on C at which the gradient of the tangent is $\frac{1}{2}$. **[5]**

The most common approach in this part was to differentiate the given equation for the circle implicitly and set $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to $\frac{1}{2}$ to obtain the linear equation $2x + y = 1$. While some candidates, after deriving this equation, made no further progress most correctly substituted this equation into the equation for *C* and solved the resulting three-term quadratic in *x*. While many obtained the correct *x*-coordinates as 7 and – 1 not all correctly obtained the corresponding *y*-coordinates. Candidates are reminded that when solving a pair of simultaneous equations with one linear and one quadratic equation, it is strongly advisable to find the second set of coordinates by substituting into the linear and not the quadratic equation.

Question 5 (a)

5 A mathematics department is designing a new emblem to place on the walls outside its classrooms. The design for the emblem is shown in the diagram below.



The emblem is modelled by the region between the x-axis and the curve with parametric equations

$x = 1 + 0.2t - \cos t, \qquad y = k\sin^2 t,$

where k is a positive constant and $0 \leqslant t \leqslant \pi$.

Lengths are in metres and the area of the emblem must be $1\,\text{m}^2$.

(a) Show that $k\int_0^{\pi}(0.2 + \sin t - 0.2\cos^2 t - \sin t\cos^2 t)\,\mathrm{d}t = 1$. **[3]**

Candidates struggled with both parts of this question with examiners noting that many candidates gave the impression that they were unsure what either part of this question required of them. Some candidates in part (a) made a correct start by finding the correct expression for $\frac{\mathrm{d}x}{\mathrm{d}t}$ but only the most able could apply $\int y\frac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t$ correctly and derive the given result.

Question 5 (b)

(b) Determine the exact value of k. **[6]**

Although there were several ways of tackling this integral (to find the exact value of *k*) it was expected that candidates would deal with each term separately (and not attempt to combine terms together). The form of the integral given to the candidates in part (a) was meant to assist (rather than hamper) their attempts to solve this part. So, to that end it was expected that candidates would tackle the integral in the following way:

$\int 0.2 + \sin t \, \mathrm{d}t = 0.2t - \cos t$ (integrate the first two terms immediately)

$\int \cos^2 t \, \mathrm{d}t = \frac{1}{2}\int 1 + \cos 2t \, \mathrm{d}t = \frac{1}{2}\left(t + \frac{1}{2}\sin 2t\right)$ (use the standard double-angle formula to integrate $\cos^2 t$)

$\int \sin t \cos^2 t \, \mathrm{d}t = -\frac{1}{3}\cos^3 t$ (use the result $\int \mathrm{f}'(x)\left(\mathrm{f}(x)\right)^n \mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$ from the formulae given on page 2 of the question paper)

The small number of candidates who did integrate each term correctly usually applied the limits correctly too and obtained the correct value for *k*.

Assessment for learning

Candidates must be familiar with all the formulae given on page 2 and 3 of the question paper as any of these results could be required during the assessment.

Question 6 (a)

6 The first, third and fourth terms of an arithmetic progression are u_1, u_3 and u_4 respectively, where

$$u_1 = 2\sin\theta, \qquad u_3 = -\sqrt{3}\cos\theta, \qquad u_4 = \frac{7}{2}\sin\theta,$$

and $\frac{1}{2}\pi < \theta < \pi$.

(a) Determine the exact value of θ. **[3]**

This part was answered particularly well with most candidates correctly setting up an equation in sine and cosine using the fact that $2(u_4 - u_3) = u_3 - u_1$. Most solved this equation correctly using tan and obtained the correct value of θ. The most common error was to misread the question as giving the first three terms of an AP and not the first, third and fourth terms.

Question 6 (b)

(b) Hence determine the value of $\sum_{r=1}^{100} u_r$. **[3]**

Most candidates used their value of θ to obtain a value for the common difference and then applied the correct formula for the sum of the first hundred terms of an AP. Occasionally candidates used the formula for the sum of a GP or incorrectly subtracted the first term from what would have been a correct answer.

Question 7 (a)

7 A car C is moving horizontally in a straight line with velocity $v\,\text{m s}^{-1}$ at time t seconds, where $v > 0$ and $t \geqslant 0$. The acceleration, $a\,\text{m s}^{-2}$, of C is modelled by the equation

$$a = v\left(\frac{8t}{7+4t^2} - \frac{1}{2}\right).$$

(a) **In this question you must show detailed reasoning.**

Find the times when the acceleration of C is zero. **[3]**

Almost all candidates correctly set the expression for the acceleration equal to zero and obtained a correct three-term quadratic equation in *t*. However, very few showed a correct method for solving this quadratic even though the question clearly said that 'detailed reasoning' must be shown. Therefore, it was common for candidates to only score 2 marks in this part.

Misconception

It is not unusual for candidates to attempt to 'reverse engineer' the factorisation from the solutions obtained using the quadratic solve function on their calculator. In this case losing a factor of 4 going from $4t^2 - 16t + 7$ to $(t - 0.5)(t - 3.5)$. For more details see the blog A Level Maths: Solving complex quadratic equations.

Exemplar 1

7(a)

$a = 0$

$0 = v\left(\frac{8t}{7+4t^2} - \frac{1}{2}\right)$

$0 = \frac{8t}{7+4t^2} - \frac{1}{2}$

$\frac{8t}{7+4t^2} = \frac{1}{2}$

$8t = \frac{1}{2}(7 + 4t^2)$

$8t = 7/2 + 2t^2$

$2t^2 - 8t + 7/2 = 0$

$4t^2 - 16t + 7 = 0$

$(t - 3.5)(t - 0.5)$

$t = 0.5\,\text{s}, \quad 3.5\,\text{s}$

Note, as this is a detailed reasoning (DR) question, the solution of the quadratic equation should demonstrate that a fully correct method has been applied (which does not rely on solving directly from a calculator). The candidate's factorised expression does not immediately follow on from their previous line of working and so only 2 marks could be awarded for this attempt.

Question 7 (b)

At $t = 0$ the velocity of C is $17.5\,\text{m s}^{-1}$ and at $t = T$ the velocity of C is $5\,\text{m s}^{-1}$.

(b) By setting up and solving a differential equation, show that T satisfies the equation

$$T = 2\ln\left(\frac{7+4T^2}{2}\right).$$ **[6]**

It was surprising in this part how many candidates did not set up a correct differential equation using the fact that $a = \frac{\mathrm{d}v}{\mathrm{d}t}$. Those that did write $\frac{\mathrm{d}v}{\mathrm{d}t} = v\left(\frac{8t}{7+4t^2} - \frac{1}{2}\right)$ usually separated the variables correctly and went on to correctly derive the given result (using the results that $v = 17.5$ when $t = 0$ and $v = 5$ when $t = T$). Some candidates, after correctly setting up the required differential equation were unsuccessful in their attempt to integrate $\int \frac{8t}{7+4t^2}\,\mathrm{d}t$ even though the result $\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)|$ is provided in the formulae at the beginning of the assessment.

Question 7 (c)

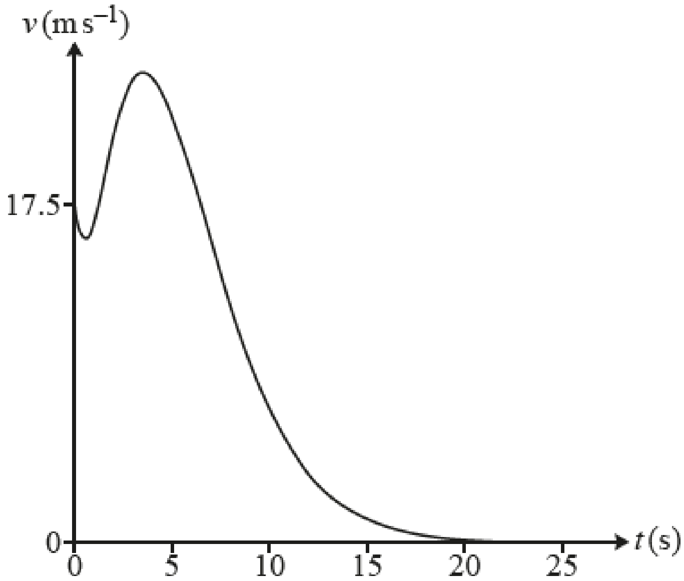
(c) Use an iterative formula, based on the equation in part **(b)**, to find the value of T, giving your answer correct to **4** significant figures. Use an initial value of 11.25 and show the result of each step of the iteration process. **[2]**

This part was answered extremely well with most candidates using the iterative equation

$T_{n+1} = 2\ln\left(\frac{7+4T_n^2}{2}\right)$ and showing sufficient iterations to obtain the correct value of 11.01 for T.
Occasionally, some candidates attempted to apply Newton-Raphson or did not give the answer to the required 4 significant figures.

Question 7 (d)

(d) The diagram below shows the velocity-time graph for the motion of C.



Find the time taken for C to decelerate from travelling at its maximum speed until it is travelling at $5\,\mathrm{m\,s^{-1}}$. **[1]**

Very few candidates used the given velocity-time graph correctly, together with the answers from parts (a) and (c) to obtain the correct answer of 7.51 in this part.

Section B overview

Two general points with regards to the answering of certain mechanics questions should be made in this overview. The first is that unless told otherwise the value that candidates should use for the acceleration due to gravity, *g*, is 9.8 and not 10 or 9.81 (and this value is stated explicitly on the front cover of the examination paper). However, in Question 12 candidates were explicitly told to take this value as 10. Secondly, when applying Newton's second law in the context of connected particles, centres (when teaching) and candidates (when answering examination questions) are strongly encouraged to apply

F = *ma* to each particle separately rather than attempting to apply this equation to the whole system. These attempts generally resulted in either the incorrect number of forces on the left-hand side of the equation or errors with the mass/acceleration of the combined system on the right-hand side. Often these attempts scored no marks (as was commonly seen in this paper in Question 13 (a)).

Question 8

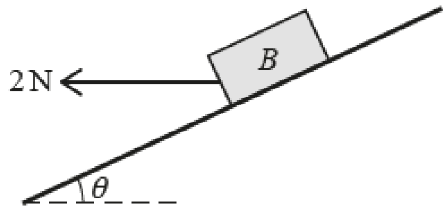
8 A particle P moves with constant acceleration $(3\mathbf{i} - 2\mathbf{j})\,\text{m s}^{-2}$. At time $t = 4$ seconds, P has velocity $6\mathbf{i}\,\text{m s}^{-1}$.

Determine the speed of P at time $t = 0$ seconds. **[4]**

Most candidates correctly applied $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ to obtain the initial velocity vector for *P* as $-6\mathbf{i} + 8\mathbf{j}$. Sadly, many candidates left their answer as a vector and did not find the corresponding speed as directed. In this case the number of marks available should have been a strong indication of the amount of work required.

Question 9 (a)

9



A block B of weight 10 N lies at rest in equilibrium on a rough plane inclined at θ to the horizontal. A horizontal force of magnitude 2 N, acting above a line of greatest slope, is applied to B (see diagram).

(a) Complete the diagram in the Printed Answer Booklet to show all the forces acting on B. **[1]**

This part was answered extremely well with most candidates correctly adding the weight and the two contact forces (normal and friction) to their diagram. Candidates are reminded that arrows should be included to show the direction of these forces. The most common error in this part was to draw the normal contact force at *B* acting vertically upwards rather than perpendicular to the plane.

Question 9 (b)

It is given that B remains at rest and the coefficient of friction between B and the plane is 0.8.

(b) Determine the greatest possible value of $\tan\theta$. **[5]**

Most candidates attempted to resolve parallel and perpendicular to the plane for *B* but they were not always successful. Aside from the usual sin/cos confusion and sign errors it was surprising how many candidates stated the correct expression for the friction component, namely, $2\cos\theta + 10\sin\theta$ but incorrectly stated the normal contact force as $10\cos\theta$ only (or vice-versa). Those candidates who did resolve in both directions correctly usually went on to apply $F = \mu R$ correctly and obtain the greatest value of $\tan\theta$. While some candidates attempted to resolve vertically and horizontally (rather than perpendicular and parallel to the plane) these responses were rarely successful.

Assessment for learning

- When resolving in two orthogonal directions candidates are reminded that if a component of a force appears in one term, then it must appear in the other term with the 'opposite' trig ratio. For example, in this question when resolving parallel to the plane the expression for the frictional force contains a component of the 2 N force $(2\cos\theta)$ and a component of the weight $(10\sin\theta)$. Therefore, when resolving perpendicular to the plane the equivalent expression for the normal contact force must also contain a component of the 2 N force $(2\sin\theta)$ and a component of the weight $(10\cos\theta)$ too.
- Candidates are reminded that they should simplify numerical expressions when giving their final answers, even if the examination question does not explicitly ask them to do so (as detailed on page 10 of the specification). So, in this question a final answer of $\tan\theta = \frac{6}{11.6}$ was not acceptable and should have been given as $\frac{15}{29}$ (exact equivalent without decimals in numerator and denominator was allowed here, $\frac{30}{58}$).

Exemplar 2

9(b)	$Fr = 10 \times \sin\theta$
	$Fr = 0.8 \times R_1$
	$R_1 + 2 \times \sin\theta = 10\cos\theta$.
	$10\sin\theta = 0.8R_1$
	$12.5\sin\theta + 2\sin\theta = 10\cos\theta$.
	$14.5\sin\theta = 10\cos\theta$.
	$\frac{14.5\sin\theta}{10\cos\theta} = \theta$.
	$\theta = 55.4$.

This response scored 2 marks. This candidate has derived a correct expression for the normal contact force but their corresponding expression for the frictional contact force only contains a component of the weight (and not the 2 N force too). As their expression for *F* does not contain the correct number of terms the second method mark for applying $F = \mu R$ could not be awarded.

Question 10 (a)

10 A particle P of mass m kg is moving on a smooth horizontal surface under the action of two constant horizontal forces $(-4\mathbf{i}+2\mathbf{j})$N and $(a\mathbf{i}+b\mathbf{j})$N. The resultant of these two forces is $\mathbf{R}$N. It is given that $\mathbf{R}$ acts in a direction which is parallel to the vector $-\mathbf{i}+3\mathbf{j}$.

(a) Show that $3a+b=10$. **[3]**

There were two common errors in this part. The first was the incorrect assumption that the resultant of the two forces was found by subtracting rather than adding the two horizontal forces together (possibly candidates were confusing 'resultant' vector with 'direction' vector). The other common error was setting the resultant of the two forces equal to $-\mathbf{i}+3\mathbf{j}$ and not a multiply of this vector. Those that did state that $\begin{pmatrix}-4+a \\ 2+b\end{pmatrix} = k\begin{pmatrix}-1 \\ 3\end{pmatrix}$ usually eliminated *k* and derived the given result correctly.

Question 10 (b)

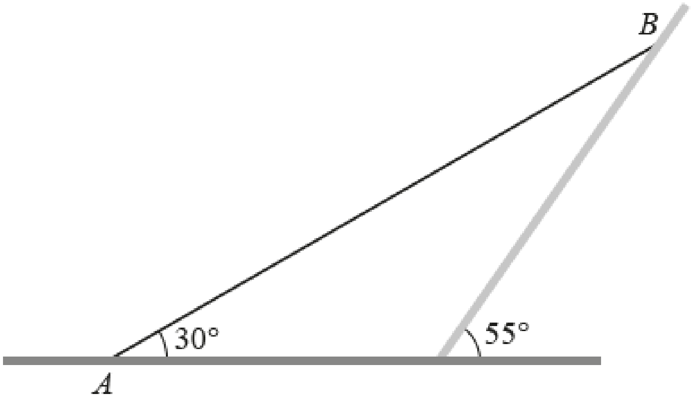
It is given that $a = 6$ and that P moves with an acceleration of magnitude $5\sqrt{10}\,\mathrm{m\,s^{-2}}$.

(b) Determine the value of m. **[4]**

This part was answered well with most candidates using the given value of *a* to then find *b* and hence determine that $\mathbf{R} = 2\mathbf{i} - 6\mathbf{j}$. Most then went on to correctly apply $|\mathbf{F}| = m|\mathbf{a}|$ and found the correct value of *m*.

Question 11 (a)

11



A uniform rod AB, of weight 20 N and length 2.8 m, rests in equilibrium with the end A in contact with rough horizontal ground and the end B resting against a smooth wall inclined at 55° to the horizontal. The rod, which rests in a vertical plane that is perpendicular to the wall, is inclined at 30° to the horizontal (see diagram).

(a) Show that the magnitude of the force acting on the rod at B is 9.56 N, correct to **3** significant figures. **[3]**

Although most candidates realised that the best way to find the magnitude of the force acting on the rod at *B* was to take moments about point *A*, many struggled with the corresponding geometry of the situation. Although most candidates correctly realised that the expression for the moment of the weight was given by $1.4(20\cos 30)$, most could not deal with the corresponding moment for the force acting at *B* with many having either an angle of 30, 60, 35 or 55.

Assessment for learning

Candidates are strongly advised when taking moments to make it clear to the examiners which point (in this case on the rod) they are taking moments about. A number of candidates attempted to take moments about another point other than *A* together with resolve forces vertically and/or horizontally; these attempts were usually unsuccessful.

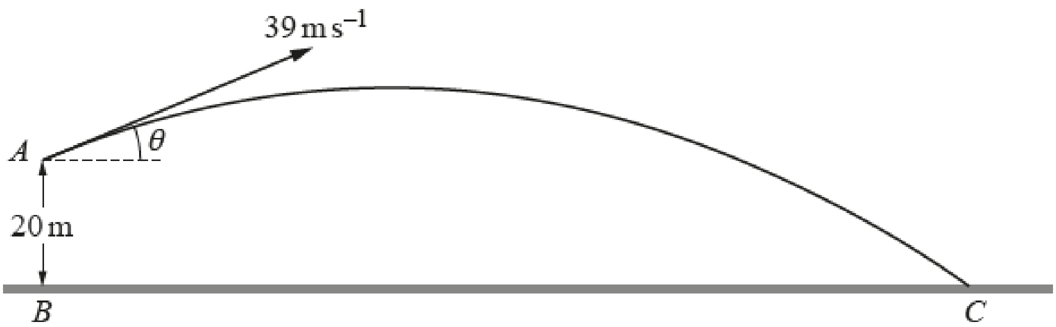
Question 11 (b)

(b) Determine the magnitude of the contact force between the rod and the ground. Give your answer correct to **3** significant figures. **[5]**

There was a common misconception here that that the 'magnitude of the contact force between the rod and the ground' was just referring to the normal contact force at the ground and not the frictional force too. The same issue in part (a) was evident in this part too as many could not deal with the geometry when resolving vertically and horizontally (again it was common to see angles of 30 and 60 being used in this part even from those candidates who had the correct angle of either 25 or 65 in part (a)). Some candidates attempted to take moments again (most commonly about *B*) or resolved parallel and perpendicular to the rod but these attempts were rarely successful (as in most cases one or more forces were missing).

Question 12 (a)

12 In this question you should take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$.



A small ball P is projected from a point A with speed $39\,\mathrm{m\,s^{-1}}$ at an angle of elevation θ, where $\sin\theta = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$. Point A is 20 m vertically above a point B on horizontal ground. The ball first lands at a point C on the horizontal ground (see diagram).

The ball P is modelled as a particle moving freely under gravity.

(a) Find the maximum height of P above the ground during its motion. **[3]**

This part was answered extremely well with most candidates taking the direct route of applying

$v^2 = u^2 + 2as$ with $v = 0, a = -10$ and $u = 39\sin\theta$. The most common errors were using $u = 39$ or $39\cos\theta$, using 9.8 for *g*, or only working out the vertical distance from A to the max. height (and so forgetting to add the distance of 20 from *A* to *B*).

Question 12 (b)

The time taken for P to travel from A to C is T seconds.

(b) Determine the value of T. **[3]**

Similarly to part (a) this part was answered extremely well with most candidates correctly applying $s = ut + \frac{1}{2}at^2$ with $s = -20$, $a = -10$ and $u = 39\sin\theta$ and then correctly using their calculator to find the required value of $T = 4$. The errors mentioned in part (a) were commonly seen in this part too.

Question 12 (c)

(c) State **one** limitation of the model, other than air resistance or the wind, that could affect the answer to part **(b)**. **[1]**

Although there were many suitable correct answers seen by examiners (for example, using a more accurate value of *g*, considering the dimensions or spin of *P*, etc.) the most common incorrect responses were those that referred to the mass or weight or shape of *P* (and in some cases mentioning air resistance, wind or the fact that the ground may not be horizontal).

Assessment for learning

When candidates are asked to give **one** written answer (in this case a limitation to a model) they should do just that. If multiple answers are given, then all must be valid to score the single mark available.

Question 12 (d)

At the instant that P is projected, a second small ball Q is released from rest at B and moves towards C along the horizontal ground.

At time t seconds, where $t \geqslant 0$, the velocity $v\,\mathrm{m\,s^{-1}}$ of Q is given by

$$v = kt^3 + 6t^2 + \tfrac{3}{2}t,$$

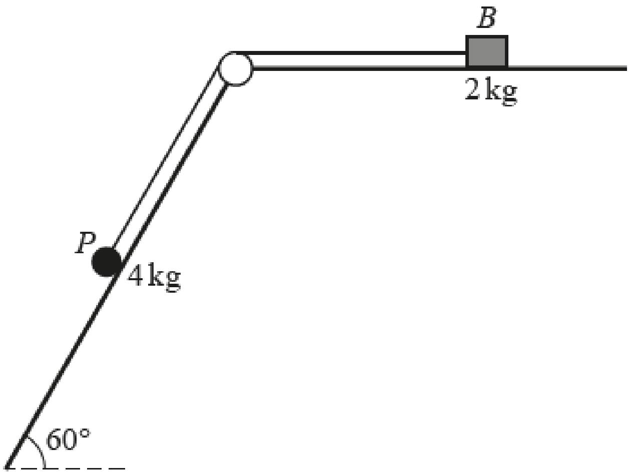
where k is a positive constant.

(d) Given that P and Q collide at C, determine the acceleration of Q immediately before this collision. **[6]**

The responses to this part were mixed with some candidates making little progress apart from differentiating the given expression for *v* to obtain an expression for *a* in terms of *t*. Those candidates who realised that they had to use the value of *T* found in part (b) to find the horizontal distance *BC* and then integrate to set up an equation to find the value of *k* were usually successful. Although not penalised in the first two parts of the question, using an incorrect value of *g* meant that neither of the 2 accuracy marks (for finding *k* and then the required acceleration) could be awarded in this part. It was surprising how many candidates used equations for constant acceleration in this part to model the motion of *Q*.

Question 13 (a)

13



The diagram shows a small block B, of mass 2 kg, and a particle P, of mass 4 kg, which are attached to the ends of a light inextensible string. The string is taut and passes over a small smooth pulley fixed at the intersection of a horizontal surface and an inclined plane. The particle can move on the inclined plane, which is rough, and which makes an angle of 60° with the horizontal. The block can move on the horizontal surface, which is also rough.

The system is released from rest, and in the subsequent motion P moves down the plane and B does not reach the pulley.

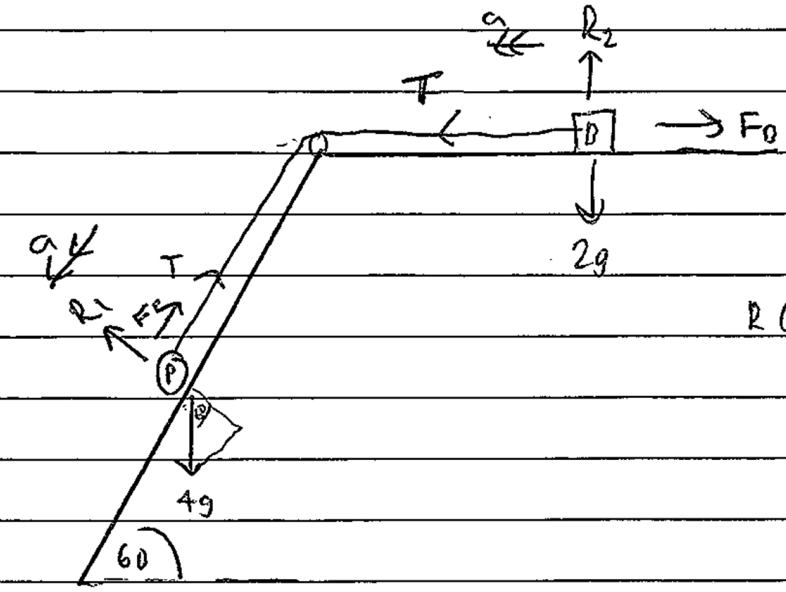
It is given that the coefficient of friction between P and the inclined plane is twice the coefficient of friction between B and the horizontal surface.

(a) Determine, in terms of g, the tension in the string. **[7]**

Most candidates in this part attempted to apply Newton's second law for the motion of *P* and *B* separately to obtain two equations relating the tension in the string, the acceleration of the system and the coefficients of friction of *P* and *B*. If done correctly this resulted in the equations: $4gsin60 - \mu_P(4gcos60) - T = 4a$ and $T - \mu_B(2g) = 2a$. It was surprising therefore how many candidates derived these two equations correctly (and so scored the first 5 marks) but did not complete the problem by eliminating a, and use the given result that $\mu_P = 2\mu_B$, to find the required tension in the string. Apart from the common sign errors and sine/cosine confusion other errors seen in this part were assuming that the acceleration of the system was zero or assuming that as $\mu_P = 2\mu_B$ then this implied that $F_P = 2F_B$ (which although true in this particular case it could not be assumed to be true without evidence of considering the relevant normal contact forces).

Exemplar 3

13(a)



$R(\uparrow): R_1 = 4g\cos 60$

$F_P = 2\mu R_1$

$R(\leftarrow): \quad T - F_B = 2a$

$F_B = \mu R_2$

$R(\uparrow): \quad R_2 = 2g$

$F_B = \mu 2g$

$R(\nearrow): \quad 4g\sin 60 - T - F_P = 4a$

$F_P = 2\mu 4g\cos 60$

$F_P = 8g\mu\cos 60$

$4g\sin 60 - F_B - F_P = 6a$

$4g\sin 30 - \mu 2g - 8g\mu\cos 60 = 6a$

$2g - 2\mu g - 4\mu g = 6a$

$2g - 6\mu g = 6a$

$a = \frac{2g}{6} - \mu g$

$T - 2\mu g = 2\left(\frac{2g}{6} - \mu g\right)$ $\quad 2\left(\frac{2g - 6\mu g}{6}\right)$

$\frac{4g - 12\mu g}{6}$

$T - 2\mu g = \frac{4g - 12\mu g}{6}$

$T = \frac{2}{3}g - 2\mu g + 2\mu g$

$T = \frac{2}{3}g$

This response scored the first 5 marks. This candidate has set their working out in a clear and logical manner (and included a diagram to assist them in setting up the required equations of motion). The candidate manages to set up both equations for *P* and *B* correctly but then makes an error when they write sin 30 rather than sin 60 which leads to an incorrect expression for the tension in the string.

Question 13 (b)

When P is moving at $2\,\mathrm{m\,s^{-1}}$ the string breaks. In the 0.5 seconds after the string breaks P moves 1.9 m down the plane.

(b) Determine the deceleration of B after the string breaks. Give your answer correct to **3** significant figures. **[5]**

This part proved challenging for the majority of candidates, with most only getting as far as calculating the acceleration of *P* as it moved down the plane. Only the highest scoring candidates set $T = 0$ in their attempt at Newton's second law for *P* (from part (a)) to obtain the coefficient of friction for *P*. Those that did this were usually successful in obtaining the correct coefficient of friction for *B* and hence its deceleration.

Supporting you

Teach Cambridge

Make sure you visit our secure website **Teach Cambridge** to find the full range of resources and support for the subjects you teach. This includes secure materials such as set assignments and exemplars, online and on-demand training.

Don't have access? If your school or college teaches any OCR qualifications, please contact your exams officer. You can forward them this link to help get you started.

Reviews of marking

If any of your students' results are not as expected, you may wish to consider one of our post-results services. For full information about the options available visit the OCR website.

Access to Scripts

For the June 2023 series, Exams Officers will be able to download copies of your candidates' completed papers or 'scripts' for all of our General Qualifications including Entry Level, GCSE and AS/A Level. Your centre can use these scripts to decide whether to request a review of marking and to support teaching and learning.

Our free, on-demand service, Access to Scripts is available via our single sign-on service, My Cambridge. Step-by-step instructions are on our website.

Keep up-to-date

We send a monthly bulletin to tell you about important updates. You can also sign up for your subject specific updates. If you haven't already, sign up here.

OCR Professional Development

Attend one of our popular CPD courses to hear directly from a senior assessor or drop in to a Q&A session. Most of our courses are delivered live via an online platform, so you can attend from any location.

Please find details for all our courses for your subject on **Teach Cambridge**. You'll also find links to our online courses on NEA marking and support.

Signed up for ExamBuilder?

ExamBuilder is the question builder platform for a range of our GCSE, A Level, Cambridge Nationals and Cambridge Technicals qualifications. Find out more.

ExamBuilder is **free for all OCR centres** with an Interchange account and gives you unlimited users per centre. We need an Interchange username to validate the identity of your centre's first user account for ExamBuilder.

If you do not have an Interchange account please contact your centre administrator (usually the Exams Officer) to request a username, or nominate an existing Interchange user in your department.

Active Results

Review students' exam performance with our free online results analysis tool. It is available for all GCSEs, AS and A Levels and Cambridge Nationals.

Find out more.

Need to get in touch?

If you ever have any questions about OCR qualifications or services (including administration, logistics and teaching) please feel free to get in touch with our customer support centre.

Call us on
01223 553998

Alternatively, you can email us on
support@ocr.org.uk

For more information visit
- **ocr.org.uk/qualifications/resource-finder**
- **ocr.org.uk**
- **facebook.com/ocrexams**
- **twitter.com/ocrexams**
- **instagram.com/ocrexaminations**
- **linkedin.com/company/ocr**
- **youtube.com/ocrexams**

We really value your feedback

Click to send us an autogenerated email about this resource. Add comments if you want to. Let us know how we can improve this resource or what else you need. Your email address will not be used or shared for any marketing purposes.

I like this **I dislike this**

Please note – web links are correct at date of publication but other websites may change over time. If you have any problems with a link you may want to navigate to that organisation's website for a direct search.

CAMBRIDGE
UNIVERSITY PRESS & ASSESSMENT