OCR
Oxford Cambridge and RSA

Wednesday 8 June 2022 – Afternoon

AS Level Mathematics B (MEI)

H630/02 Pure Mathematics and Statistics

Time allowed: 1 hour 30 minutes

6120200295

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Answer **all** the questions.

1 The probability distribution for the discrete random variable X is shown below.

x	1	2	3	4	5
$P(X = x)$	0.2	0.15	a	0.27	0.14

Find the value of a. **[1]**

2 **(a)** Factorise $3x^2 - 19x - 14$. **[2]**

(b) Solve the inequality $3x^2 - 19x - 14 < 0$. **[2]**

3 You are given that $y = Ae^{0.02t}$.

- Make t the subject of the formula.
- Find the value of t when $y = 10^8$ and $A = 6.62 \times 10^7$. **[3]**

4 The position vector of P is $\mathbf{p} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and the position vector of Q is $\mathbf{q} = \begin{pmatrix} 28 \\ 10 \end{pmatrix}$.

(a) Determine the magnitude of $\overrightarrow{PQ}$. **[2]**

(b) Determine the angle between $\overrightarrow{PQ}$ and the positive x-direction. **[2]**

5 Ali collected data from a random sample of 200 workers and recorded the number of days they each worked from home in the second week of September 2019. These data are shown in **Fig. 5.1**.

Number of days worked from home	0	1	2	3	4	5
Frequency	41	65	33	26	20	15

Fig. 5.1

(a) Represent the data by a suitable diagram. **[2]**

(b) Calculate

- The mean number of days worked from home.
- The standard deviation of the number of days worked from home. **[2]**

Ali then collected data from a different random sample of 200 workers for the same week in September 2019. The mean number of days worked from home for this sample was 1.94 and the standard deviation was 1.75.

(c) Explain whether there is any evidence to suggest that one or both of the samples must be flawed. **[1]**

Fig. 5.2 shows a cumulative frequency diagram for the ages of the workers in the first sample who worked from home on at least one day.

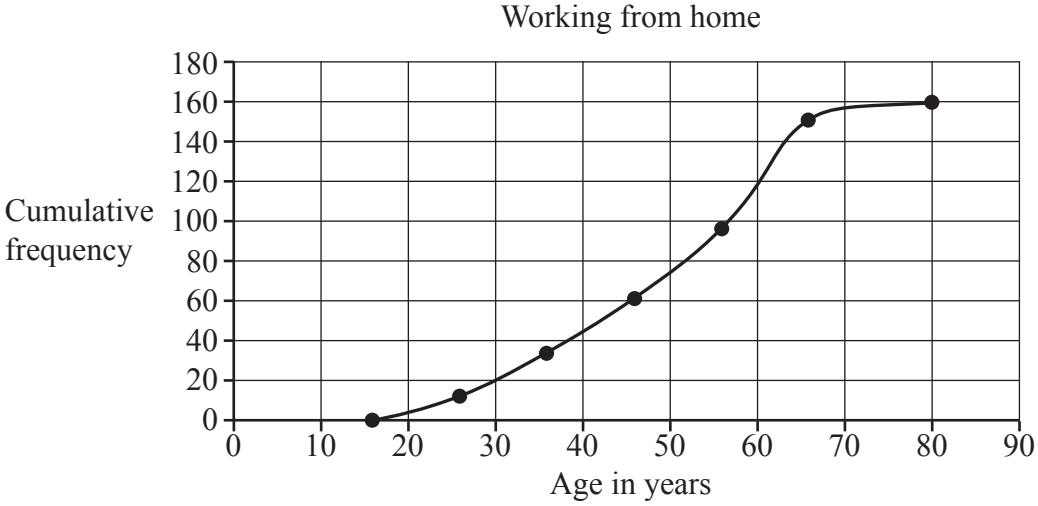
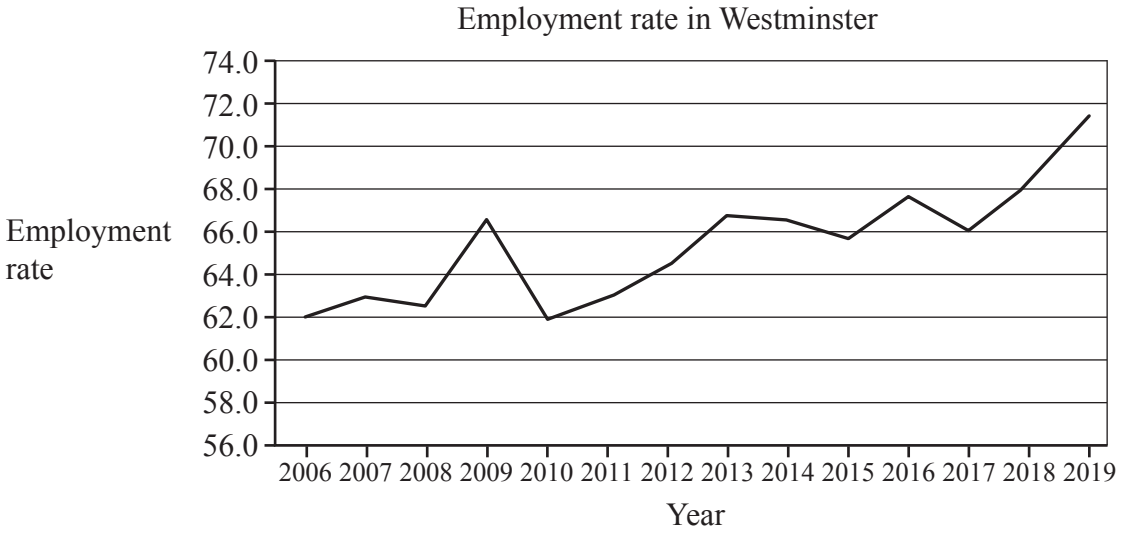


Fig. 5.2

Ali concludes that 90% of the workers in this sample who worked from home on at least one day were under 60 years of age.

(d) Explain whether Ali's conclusion is correct. **[1]**

6 The pre-release material contains information about employment rates in London boroughs. The graph shows employment rates for Westminster between 2006 and 2019.



A local politician stated that the diagram shows that more than 60% of seventy-year-olds were in employment throughout the period from 2006 to 2019.

(a) Use your knowledge of the pre-release material to explain whether there is any evidence to support this statement. **[1]**

In order to estimate the employment rate in 2020, two different models were proposed using the LINEST function in a spreadsheet.

Model 1 (using all the data from 2006 onwards)
$Y = 0.549x - 1040$,

Model 2 (using data from 2017 onwards)
$Y = 2.65x - 5280$,

where Y = employment rate and x = calendar year.

It was subsequently found that the employment rate in Westminster in 2020 was 68.4%.

(b) Determine which of the two models provided the better estimate for the employment rate in Westminster in 2020. **[3]**

(c) Use your knowledge of the pre-release material to explain whether it would be appropriate to use either model to estimate the employment rate in 2020 in other London boroughs. **[1]**

(d) What does model 2 predict for employment rates in Westminster in the long term? **[1]**

7 **(a)** On the pair of axes in the Printed Answer Booklet, sketch the graphs of

- $y = 2x + 4$
- $y = \frac{2}{x}$ **[2]**

(b) Determine the x-coordinates of the points of intersection of the line $y = 2x + 4$ and the curve $y = \frac{2}{x}$, giving your answers in an exact form. **[5]**

8 In 2018 research showed that 81% of young adults in England had never donated blood.

Following an advertising campaign in 2021, it is believed that the percentage of young adults in England who had never donated blood in 2021 is less than 81%.

Ling decides to carry out a hypothesis test at the 5% level.

Ling collects data from a random sample of 400 young adults in England.

(a) State the null and alternative hypotheses for the test, defining the parameter used. **[2]**

(b) Write down the probability that the null hypothesis is rejected when it should in fact be accepted. **[1]**

(c) Assuming the null hypothesis is correct, calculate the expected number of young adults in the sample who had never donated blood. **[1]**

(d) Calculate the probability that there were no more than 308 young adults who had never donated blood in the sample. **[1]**

(e) Determine the critical region for the test. **[3]**

In fact, the sample contained 314 young adults who had never donated blood.

(f) Carry out the test, giving the conclusion in the context of the question. **[3]**

9 The equation of a curve is $y = 12x - 4x^{\frac{3}{2}}$.

(a) State the coordinates of the intersection of the curve with the y-axis. **[1]**

(b) Find the value of y when $x = 9$. **[1]**

(c) Determine the coordinates of the stationary point. **[5]**

(d) Sketch the curve, giving the coordinates of the stationary point and of any intercepts with the axes. **[3]**

10 In this question you must show detailed reasoning.

The equation of a curve is $y = 12x^3 - 24x^2 - 60x + 72$.

Determine the magnitude of the total area bounded by the curve and the x-axis. **[9]**

11 The pre-release material contains information about the Median Income of Taxpayers and the Percentage of Pupils Achieving at Least 5 A*– C grades, including English and Maths, at the end of KS4 in different areas of London.

Alex is investigating whether there is a relationship between median income and the percentage of pupils achieving at least 5 A* – C grades, including English and Maths, at the end of KS4. Alex decides to use the first 12 rows of data for 2014–5 from the pre-release data as a sample. The sample is shown in **Fig. 11.1**.

Area	Median Income of Taxpayers	Percentage of Pupils Achieving at Least 5 A*– C grades including English and Maths
City of London	61100	#N/A
Barking and Dagenham	21800	54.0
Barnet	27100	70.1
Bexley	24400	55.0
Brent	22700	60.0
Bromley	28100	68.0
Camden	33100	56.4
Croydon	25100	59.6
Ealing	24600	62.1
Enfield	25300	54.5
Greenwich	24600	57.7
Hackney	26000	60.4

Fig. 11.1

(a) Explain whether the data in **Fig. 11.1** is a simple random sample of the data for 2014–5. **[1]**

(b) The City of London is included in Alex's sample.

Explain why Alex is not able to use the data for the City of London in this investigation. **[2]**

Fig. 11.2 shows a scatter diagram showing Percentage of Pupils against Median Income for all of the areas of London for which data is available.

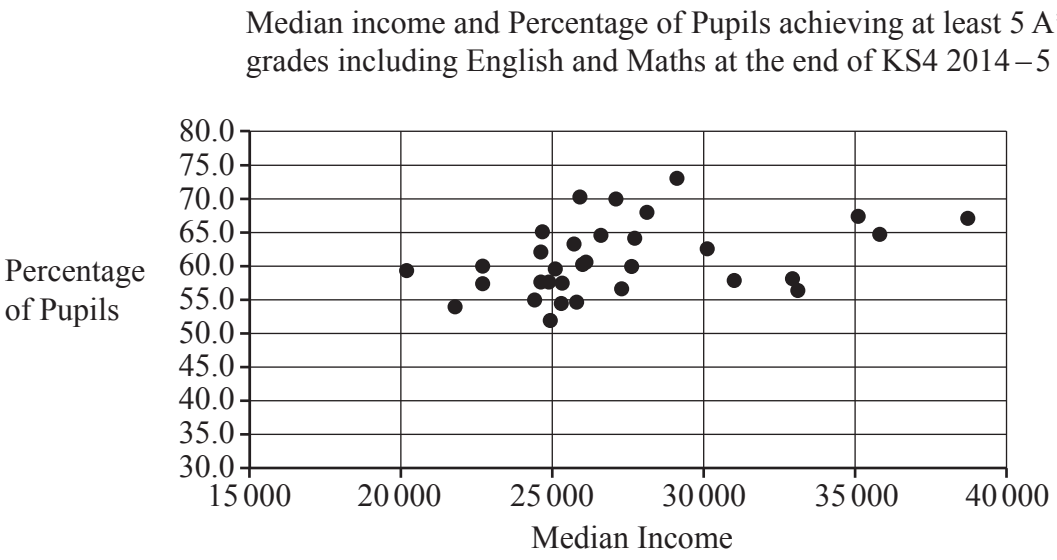


Fig. 11.2

Alex identifies some outliers.

(c) On the copy of **Fig. 11.2** in the Printed Answer Booklet, ring three of these outliers. **[1]**

Alex then discards all the outliers and uses the LINEST function on a spreadsheet to obtain the following model.

$P = 0.0009049M + 37.38$,
where P = percentage of pupils and M = median income.

(d) Show that the model is a good fit for the data for Hackney. **[1]**

(e) Use the model to find an estimate of the value of P for City of London. **[1]**

(f) Give **two** reasons why this estimate may not be reliable. **[2]**

Alex states that more than 50% of the pupils in London achieved at least a grade C at the end of KS4 in English **and** Maths in 2014–5.

(g) Use the information in **Fig. 11.2** together with your knowledge of the pre-release material to explain whether there is evidence to support this statement. **[1]**

END OF QUESTION PAPER

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