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AS LEVEL

Examiners' report

MATHEMATICS A

H230
For first teaching in 2017

H230/02 Summer 2019 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper 2 series overview

This is the second series of the reformed decoupled linear AS Maths specification. H230/02 is the second of the two examination units for the AS Level examination for GCE Mathematics A. The exam is structured in two sections. Section A : Pure Mathematics (this paper consists of 8 questions allocated 49 marks) and Section B : Mechanics (this paper consists of three questions allocated 26 marks). All questions should be answered. Each section has a gradient of difficulty throughout the section and consists of a mixture of short and long questions. Consequently, some candidates would be well advised to attempt the first few (easier) Pure Maths questions and the first few (easier) Mechanics questions, before attempting the more difficult questions in each section.

Three overarching themes are applied across all content.

1. Mathematical argument, language and proof,

2 Mathematical problem solving.

3. Mathematical modelling.

To do well on this paper candidates need to be comfortable applying their knowledge and understanding to all three of these overarching themes, in both familiar and unfamiliar contexts.

Candidate Performance Overview

Candidates who did well on this paper generally did the following.

- Performed calculations to an appropriate degree of accuracy
- Demonstrated a competent algebraic technique
- Readily identified required methods
- Coped well with unstructured questions, e.g. 7
- Interpreted modelling situations well.

Candidates who did less well on this paper generally did the following.

- Made careless mistakes in calculations
- Lacked accuracy in their algebraic work, e.g. were prone to sign errors
- Did not understand the significance of the 'command words'
- Found it difficult to develop a suitable method to solve an unstructured question
- Found it difficult to apply what they had learnt to unfamiliar situations, e.g. 5(c).

Teachers and candidates are encouraged to study carefully the requirements we expect from the defined 'command words' used in questions such as 'Determine', 'Show that', 'Hence', '**In this question you must show detailed reasoning**', etc. These are fully explained in the Specification Document.

i	**OCR support**	A poster detailing the different command words and what they mean is available here: https://teach.ocr.org.uk/italladdsup

There was no evidence that time constraints had led to a candidate underperforming.

Section A overview

Question 1

1 In this question you must show detailed reasoning.

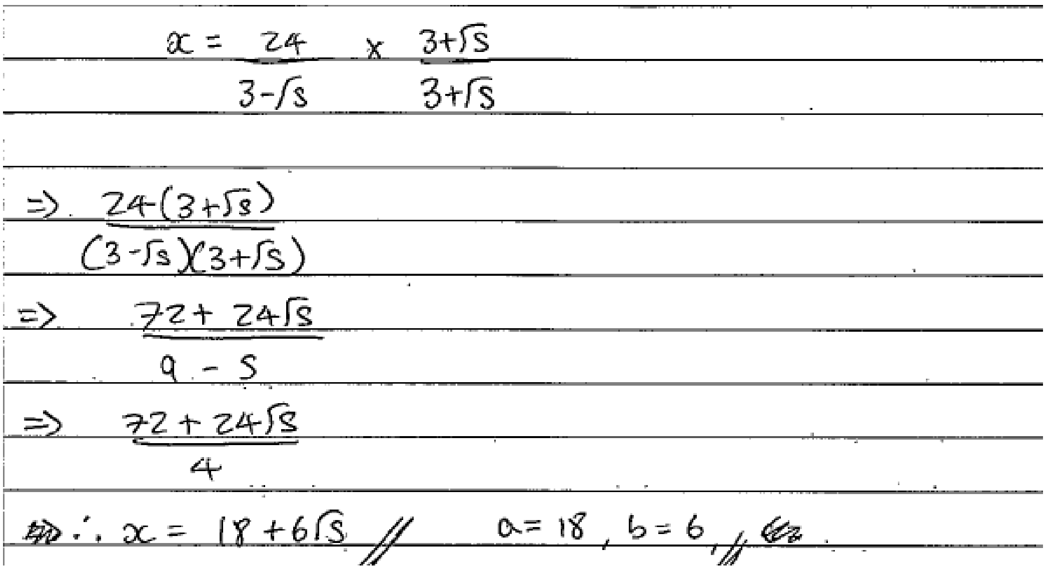
Solve the equation $x(3-\sqrt{5}) = 24$, giving your answer in the form $a+b\sqrt{5}$, where a and b are positive integers. **[3]**

This question required detailed reasoning so candidates were expected to show full working to indicate how their final answer had been obtained. The answer needed to be in the form requested.

Those who went straight to the answer without intermediate steps had clearly just used their calculator and scored 0/3.

Candidates need to be aware that detailed reasoning questions will want substantial evidence of methods.

Exemplar 1


$x = \frac{24}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$

$\Rightarrow \frac{24(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})}$

$\Rightarrow \frac{72+24\sqrt{5}}{9-5}$

$\Rightarrow \frac{72+24\sqrt{5}}{4}$

$\therefore x = 18+6\sqrt{5}$ // $a = 18,\ b = 6$ //

This candidate has made clear each step of their solution for full credit.

Question 2 (a)

2 **(a)** Express $5x^2 - 20x + 3$ in the form $p(x+q)^2 + r$, where p, q and r are integers. **[3]**

This was frequently well done, with errors in r being most common, e.g. $5(x-2)^2 - 4 + 3$. We occasionally had attempts involving $(x-10)^2$.

	AfL	The use of calculators to check answers should be encouraged. The quadratic solve function or finding the minimum point on a plotted graph would have confirmed the values for q and r. Notice that there was no specific command word used in the question to indicate any working had to be seen; candidates that knew how to obtain the completed square form from their calculator would score full marks here with correct answer only.

(b) State the coordinates of the minimum point of the curve $y = 5x^2 - 20x + 3$. **[2]**

Question 2 (b)

'State' here really should indicate to candidates that the answer can just be written down from what they have already done, but many resorted successfully to differentiation.

Question 2 (c)

(c) State the equation of the normal to the curve $y = 5x^2 - 20x + 3$ at its minimum point. **[1]**

'State' again, with the same implication as in 2(c).

This part was the least successful. A significant number of candidates found the tangent $y = -17$ rather than the requested normal $x = 2$.

Question 3 (a)

3 **(a)** Sketch the curve $y=-\frac{1}{x^2}$. **[1]**

'Sketch' is defined in the 'command words', emphasising the features we were looking for here. Candidates should be encouraged to take care with their sketches and try to be neat, with curves carefully drawn approaching the asymptotes. Given that calculators are available to check the basic shape (either directly plotting or using the table function), it was surprising to see $y=x^{-1}$ appearing so often. $y=x^2$ was also occasionally seen.

Question 3 (b)

(b) The curve $y=-\frac{1}{x^2}$ is translated by 2 units in the positive x-direction.

State the equation of the curve after it has been translated. **[2]**

This part often went wrong. We expected to see the equation '$y=$ ', not just '$\mathrm{f}(x-2)=$', and we expected the minus sign to be present for the method mark.

Many solutions involved $-\frac{1}{(x^2-2)}$.

(c) The curve $y=-\frac{1}{x^2}$ is stretched parallel to the y-axis with scale factor $\frac{1}{2}$ and, as a result, the point $\left(\frac{1}{2},-4\right)$ on the curve is transformed to the point P.

State the coordinates of P. **[2]**

Question 3 (c)

Reasonably well answered. The most common error was for candidates to state the coordinates $\left(\frac{1}{4},-2\right)$.

Question 4 (a)

4 **(a)** Find and simplify the first three terms in the expansion of $(2-5x)^5$ in ascending powers of x. **[3]**

The expansion method was done well in general, although a few used $(5x)$ rather than $(-5x)$.

A few unnecessarily tried to rewrite as $2^5\left(1-\frac{5}{2x}\right)^5$ with mixed success.

Question 4 (b)

(b) In the expansion of $(1+ax)^2(2-5x)^5$, the coefficient of x is 48.

Find the value of a. **[3]**

This part was found to be much more challenging. We wanted to see evidence of the expansion of $(1+ax)^2$ and the multiplication to give two relevant terms in x, then connected to the given 48, but this requirement was not always recognised.

Question 5 (a)

5 Points A, B, C and D have position vectors $\mathbf{a}=\begin{pmatrix}1\\2\end{pmatrix}$, $\mathbf{b}=\begin{pmatrix}3\\5\end{pmatrix}$, $\mathbf{c}=\begin{pmatrix}7\\4\end{pmatrix}$ and $\mathbf{d}=\begin{pmatrix}4\\k\end{pmatrix}$.

(a) Find the value of k for which D is the midpoint of AC. **[1]**

Very well attempted.

Question 5 (b)

(b) Find the two values of k for which $|\overrightarrow{AD}|=\sqrt{13}$. **[3]**

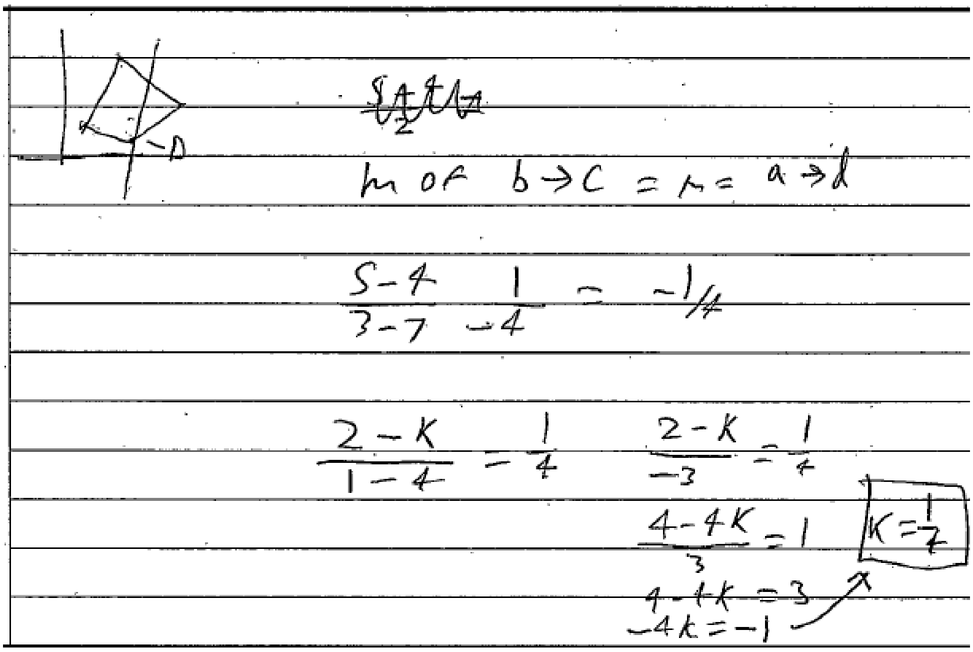
Many candidates made progress here, successfully setting up the required quadratic in k. Although $(k-2)^2=4$ provides a neat solution, more often than not the correct answers followed from $k^2-4k=0$. Some expanded the brackets incorrectly and a few candidates thought $\mathbf{AD}=\mathbf{a}+\mathbf{d}$.

Question 5 (c)

(c) Find one value of k for which the four points form a trapezium. **[2]**

This was found to be more challenging. Many had no idea how to start or produced a wrong value of k with no evidence of how it had been obtained, but those who made sensible attempts used either gradients or vectors. For those using gradients (more common), some seemingly used perpendicularity. Attention to detail was important.

Exemplar 2


m of b → c = m = a → d

$\frac{5-4}{3-7}$ $\frac{1}{-4}$ = $-1/4$

$\frac{2-k}{1-4} = \frac{1}{4}$ $\frac{2-k}{-3} = \frac{1}{4}$

$\frac{4-4k}{3} = 1$ $k = \frac{1}{4}$

$4-4k = 3$

$-4k = -1$

A common issue throughout this paper involved marks dropped due to careless notation. This candidate has shown sufficient working for the method mark, but having correctly found $m = -\frac{1}{4}$, went on to use $m = \frac{1}{4}$ and so dropped the accuracy mark.

Question 6 (a)

6 In this question you must show detailed reasoning.

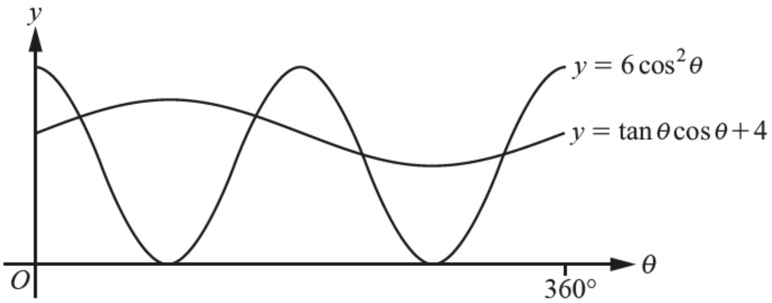
(a) Show that the equation $6\cos^2\theta = \tan\theta\cos\theta + 4$

can be expressed in the form $6\sin^2\theta + \sin\theta - 2 = 0$. **[2]**

Another question requiring detailed reasoning, so examiners scrutinised the notation carefully and expected precision in presentation. For example, $\cos\theta^2$ or just $\cos$ was penalised as was the omission of '$=0$' in the final answer. To score fully candidates needed to make explicit use of trigonometric identities, for example just replacing $\tan\theta\cos\theta$ with $\sin\theta$ without justification of $\frac{\sin\theta}{\cos\theta}\cos\theta$ was not considered sufficient reasoning.

Question 6 (b)

(b)

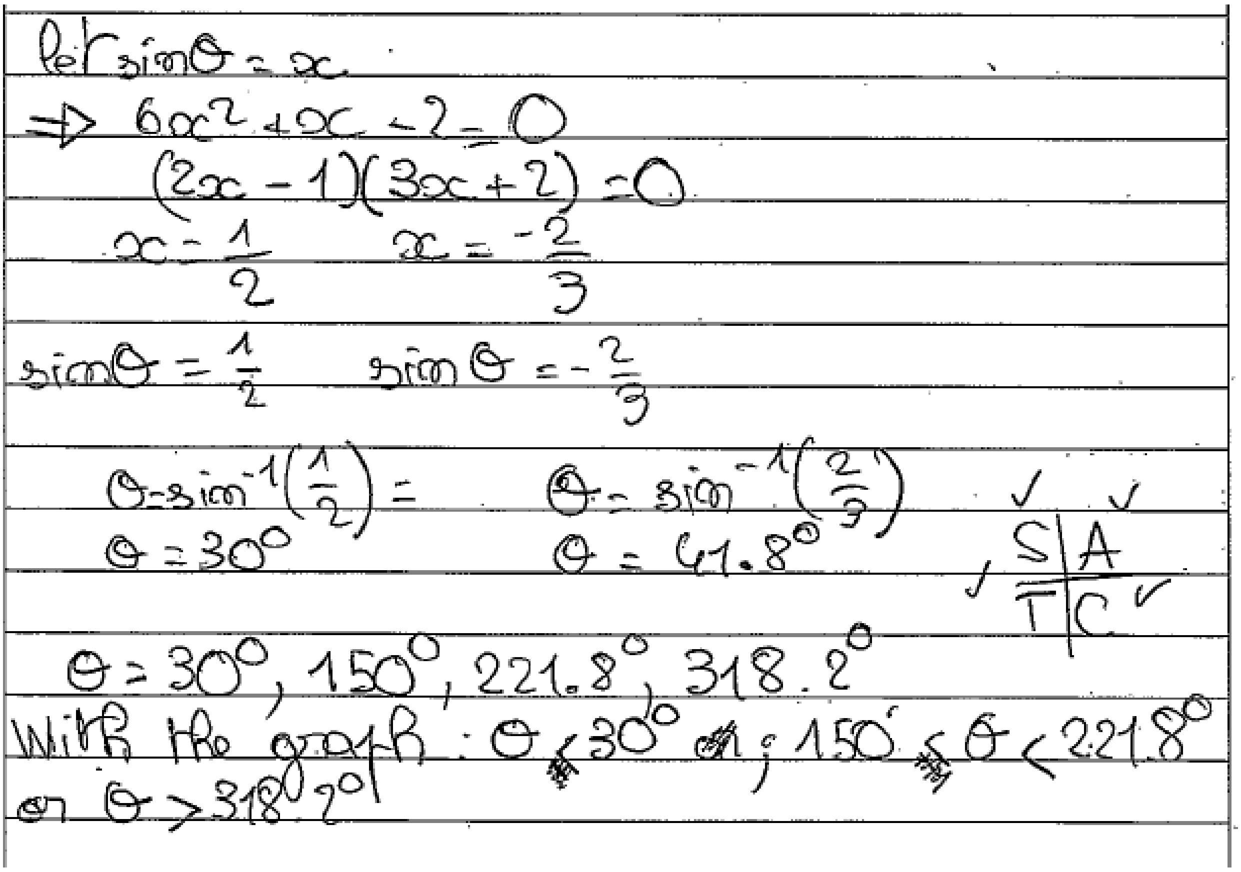


The diagram shows parts of the curves $y = 6\cos^2\theta$ and $y = \tan\theta\cos\theta + 4$, where θ is in degrees.

Solve the inequality $6\cos^2\theta > \tan\theta\cos\theta + 4$ for $0° < \theta < 360°$. **[5]**

A fully correct solution was rare. Many candidates recognised the quadratic but just found values for θ and made no effort to relate them to the given graph and form inequalities. Because this is a detailed reasoning question we expected to see how the quadratic was solved and penalised those who just produced the two roots from their calculator. Often use of inequality signs was slack and recognising which quadrants angles need to be in was sometimes confused.

Exemplar 3



Let $\sin\theta = x$

$\Rightarrow 6x^2 + x - 2 = 0$

$(2x-1)(3x+2) = 0$

$x = \frac{1}{2}$ $\quad x = -\frac{2}{3}$

$\sin\theta = \frac{1}{2}$ $\quad \sin\theta = -\frac{2}{3}$

$\theta = \sin^{-1}\left(\frac{1}{2}\right) =$ $\quad \theta = \sin^{-1}\left(\frac{2}{3}\right)$

$\theta = 30^\circ$ $\quad \theta = 41.8^\circ$

S | A
T | C

$\theta = 30^\circ, 150^\circ, 221.8^\circ, 318.2^\circ$

With the graph: $\theta < 30^\circ$; $150^\circ < \theta < 221.8^\circ$ or $\theta > 318.2^\circ$

Notice that the 'detailed reasoning' does not restrict the use of the quadratic solve function, in this exemplar the candidate has 'Let $\sin\theta = X$' with $6X^2 + X - 2 = 0$ and the substitution of the roots back in terms of θ to provide the required reasoning for this part of their answer. This candidate avoided any potential inequality pitfalls when identifying the required three regions by utilising the initial graph provided in the question.

Exemplar 4

$6\cos^2\theta > \tan\theta\cos\theta + 4$

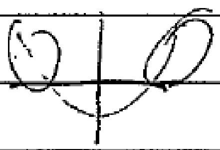
$6\sin^2\theta + \sin\theta - 2 > 0$

let $\sin\theta = x$

$6x^2 + x - 2 > 0$

$6x^2 + x - 2 = 0$

$x = \frac{1}{2}$ or $-\frac{2}{3}$



$x < -\frac{2}{3}$ or $x > \frac{1}{2}$

$\sin\theta < -\frac{2}{3}$ or $\sin\theta > \frac{1}{2}$

$\theta < -41.81$ or $\theta > 30$

S | A
T | C

$180 - -41.81$

$= 221.81°$

$360 - 41.81$

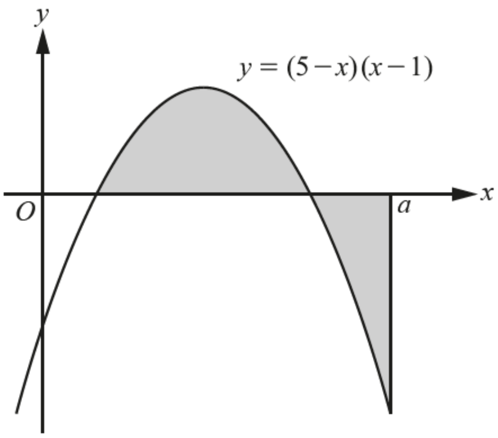
$= 318.19°$

$\theta < 221.81$ or $\theta < 318.19°$ or $\theta > 30$ or $\theta > 150°$

As in many cases, the subsequent marks are not achieved due to confusions with the inequalities.

Question 7

7



The diagram shows part of the curve $y = (5-x)(x-1)$ and the line $x = a$.

Given that the total area of the regions shaded in the diagram is 19 units^2, determine the exact value of a. **[8]**

For a fully successful attempt, candidates needed to demonstrate an understanding that areas were both above and below the *x*-axis, with the sign consequences, and algebraic accuracy was vital. Although the integral between 1 and 5 could be done on the calculator, many did not do this and evaluated it incorrectly, often because $(5-x)(x-1)$ was expanded incorrectly. Then the integral between 5 and *a* needed to be carefully done before putting everything together with the correct signs. Many cubics were wrong, so getting an exact solution proved impossible. Some obtained the correct cubic fortuitously from wrong working (which was penalised) but if all was fine by this stage we then wanted a clear explanation for the rejection of the negative root and an exact answer as requested.

Exemplar 5

$\int (5-x)(x-1)\,dx \qquad (5-x)(x-1) = 5x - 5 - x^2 + x$

$= -x^2 + 6x - 5$

$\int -x^2 + 6x - 5 \, dx$

$= \frac{-x^3}{3} + \frac{6x^2}{2} - 5x + c$

$= \left[-\frac{1}{3}x^3 + 3x^2 - 5x\right]$

where $y = 0$, $\quad -x^2 + 6x - 5 = 0$

$x = 5, 1$

$\left[-\frac{1}{3}x^3 + 3x^2 - 5x\right]_1^5$

$\left(-\frac{1}{3} \times 5^3\right) + \left(3 \times 5^2\right) - \left(5 \times 5\right) = \frac{25}{3}$

$\left(-\frac{1}{3} \times 1^3\right) + \left(3 \times 1^2\right) - \left(5 \times 1\right) = -\frac{7}{3}$

$\frac{25}{3} - -\frac{7}{3} = \frac{32}{3}$

$\frac{32}{3} + B = 19 \qquad B = \frac{25}{3}$

$\left[-\frac{1}{3}x^3 + 3x^2 - 5x\right]_5^a = \frac{25}{3}$

$\left(\frac{-a^3}{3} + 3a^2 - 5a\right) - \frac{25}{3} = \frac{25}{3}$

$\frac{-a^3}{3} + 3a^2 - 5a - \frac{50}{3} = 0$

$a = -1.577801293$

Exemplar 5 is a typical solution, failing because the signs have not been appreciated and/or the request for an exact answer ignored.

Question 8 (a)

8 **(a)** Show that the equation $2\log_2 x = \log_2(kx-1)+3$, where k is a constant, can be expressed in the form $x^2 - 8kx + 8 = 0$. **[4]**

Many candidates here scored only one mark for $\log_2 x^2$. A confidence with using the laws of logarithms was not often seen, e.g. $\log_2\left(x^2 - kx + 1\right)$ regularly stated. A few introduced 3^2 rather than 2^3. This was 'Show that' question, with the answer given in the question, so we expected sufficient justification in the written response.

Question 8 (b)

(b) Given that the equation $2\log_2 x = \log_2(kx-1)+3$ has only one real root, find the value of this root. **[4]**

Many recognised the need for '$b^2 - 4ac$', but this was sometimes carelessly applied to give $64k - 32 = 0$, $-8k^2 - 32 = 0$ or some such, and many more accurate tries only gave the positive value of k. Often the attempt stopped at the value for k with no effort to find x. Presumably candidates thought k was the root. We expected a valid reason for why the negative value for x was rejected. Simply stating $x > 0$ was not sufficient as they needed to explain that $\log_2 x$ is only defined for $x > 0$ (not that $\log_2 x$ cannot be negative).

Section B overview

Question 9

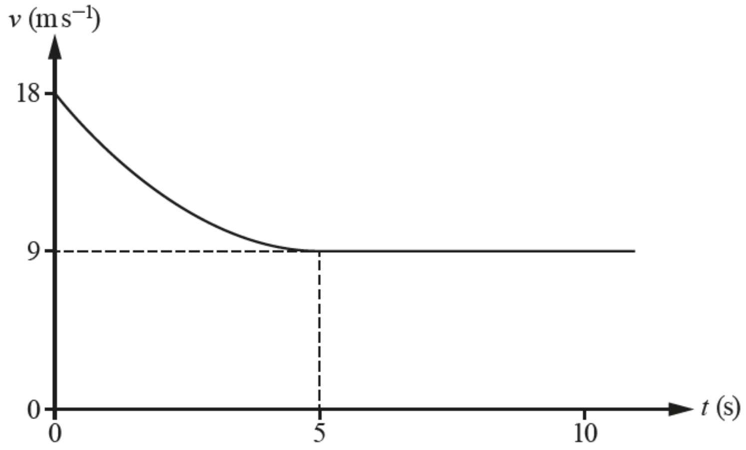
9 Three forces $\begin{pmatrix} 7 \\ -6 \end{pmatrix}$N, $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$N and $\mathbf{F}$N act on a particle.

Given that the particle is in equilibrium under the action of these three forces, calculate $\mathbf{F}$. **[2]**

This was generally very well answered.

Question 10 (a)

10



The diagram shows the velocity-time graph modelling the velocity of a car as it approaches, and drives through, a residential area.

The velocity of the car, v m s^{-1}, at time t seconds for the time interval $0 \leqslant t \leqslant 5$ is modelled by the equation $v = pt^2 + qt + r$, where p, q and r are constants.

It is given that the acceleration of the car is zero at $t = 5$ and the speed of the car then remains constant.

(a) Determine the values of p, q and r. **[5]**

Many candidates managed to gain some credit on this question. $r = 18$ was often spotted and $v = 9$ at $t = 5$ well applied. Some then stated $v = 9$ at $t = 10$ also, and floundered. Pleasingly, those who realised they needed the acceleration generally differentiated and were accurate.

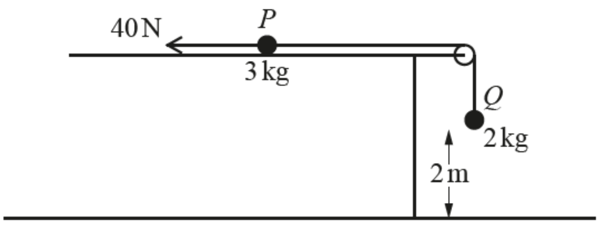
Question 10 (b)

(b) Calculate the distance travelled by the car from $t = 2$ to $t = 10$. **[3]**

The most common fundamental error here was to see an integral evaluated using the limits 0 and 10. There was no reason for candidates not to use the numerical integration function on their calculator, but more often than not showed the algebra. Without values for p, q and r found in (a) only B1 was possible.

Question 11 (a)

11 Two small balls P and Q have masses 3 kg and 2 kg respectively. The balls are attached to the ends of a string. P is held at rest on a rough horizontal surface. The string passes over a pulley which is fixed at the edge of the surface. Q hangs vertically below the pulley at a height of 2 m above a horizontal floor.



The system is initially at rest with the string taut. A horizontal force of magnitude 40 N acts on P as shown in the diagram.

P is released and moves directly away from the pulley. A constant frictional force of magnitude 8 N opposes the motion of P. It is given that P does not leave the horizontal surface and that Q does not reach the pulley in the subsequent motion.

The balls are modelled as particles, the pulley is modelled as being small and smooth, and the string is modelled as being light and inextensible.

(a) Show that the magnitude of the acceleration of each particle is $2.48\,\text{m}\,\text{s}^{-2}$. **[5]**

Candidates should be completely discouraged from treating the system as a whole (this is a high risk strategy because when thing go wrong it nearly always scores 0) and keep things simple by applying $F = ma$ to each particle. It is good practice to draw a diagram of the forces involved for each particle and show the direction of positive acceleration. This helps to avoid missing forces in the two equations, examiners considered this quite a serious error, and also encourages sensible consistent use of signs. Use of *mga* also lost the method marks. If - 2.48 appeared we expected the negative to be explained.

Question 11 (b)

(b) Find the tension in the string. **[2]**

We expected 2.48 to be used here. $2g - T = 2a$ with this value gained the method mark.

Question 11 (c)

When the balls have been in motion for 0.5 seconds, the string breaks.

(c) Find the additional time that elapses until Q hits the floor. **[5]**

This part proved challenging.
Those trying to follow the main approach on the Mark Scheme who realised they had to find the extra distance travelled until the string broke, and the velocity at this instant, then often did not take account of the signs needed for a correct equation from $s = ut + ½at^2$.Some used s = 1.69 and/or u = 0, both of which scored M0. Those who tried to break the motion into 'up' and 'down' often used 2.31as the distance down, failing to calculate the extra distance needed to reach the highest point. This approach was generally not very successful.

Question 11 (d)

(d) Find the speed of Q as it hits the floor. **[2]**

This was also spoiled by the use of u =0, s = 1.69 or even s = 2.

Question 11 (e)

(e) Write down the magnitude of the normal reaction force acting on Q when Q has come to rest on the floor. **[1]**

Well answered.

Question 11 (f)

(f) State one improvement that could be made to the model. **[1]**

Quite a few candidates suggested changes to the physical model rather than the mathematical one. So, we saw comments like: ' have a stronger string so that it does not break' or 'increase the height of the pulley'. Many thought that an improvement would be to make the surface smooth. If they suggested accounting for other external forces we wanted them to specify what force(s) they meant.

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