OCR
Oxford Cambridge and RSA

Wednesday 15 May 2019 – Morning

AS Level Mathematics B (MEI)

H630/01 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Answer **all** the questions.

1 **In this question you must show detailed reasoning.**

Show that the equation $x = 7 + 2x^2$ has no real roots. **[3]**

2 **In this question you must show detailed reasoning.**

Fig. 2 shows the graphs of $y = 4\sin x°$ and $y = 3\cos x°$ for $0 \leqslant x \leqslant 360$.

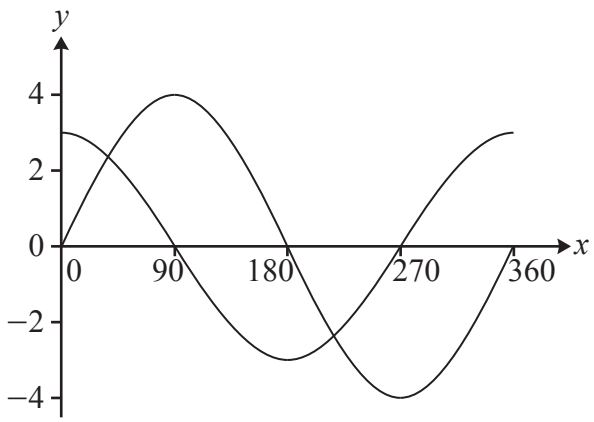

Fig. 2

Find the x-coordinates of the two points of intersection, giving your answers correct to 1 decimal place. **[3]**

3 Given that k is an integer, express $\dfrac{3\sqrt{2}-k}{\sqrt{8}+1}$ in the form $a+b\sqrt{2}$ where a and b are rational expressions in terms of k. **[4]**

4 A triangle ABC has sides AB = 5 cm, AC = 9 cm and BC = 10 cm.

(a) Find the cosine of angle BAC, giving your answer as a fraction in its lowest terms. **[2]**

(b) Find the exact area of the triangle. **[3]**

5 In this question, the unit vectors **i** and **j** are horizontal and vertically upwards respectively.

A particle has mass 2.5 kg.

(a) Write the weight of the particle as a vector. **[1]**

The particle moves under the action of its weight and two external forces $(3\mathbf{i}-2\mathbf{j})$ N and $(-\mathbf{i}+18\mathbf{j})$ N.

(b) Find the acceleration of the particle, giving your answer in vector form. **[2]**

6 Fig. 6 shows a train consisting of an engine of mass 80 tonnes pulling two trucks each of mass 25 tonnes.

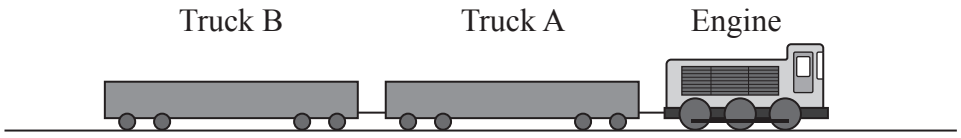

Fig. 6

The engine exerts a driving force of D N and experiences a resistance to motion of 2000 N. Each truck experiences a resistance of 600 N. The train travels in a straight line on a level track with an acceleration of $0.1\,\text{m s}^{-2}$.

(a) Complete the force diagram in the Printed Answer Booklet to show all the forces acting on the engine and each of the trucks. **[3]**

(b) Calculate the value of D. **[2]**

(c) The tension in the coupling between the engine and truck A is larger than that in the coupling between the trucks. Determine how much larger. **[2]**

7 In this question you must show detailed reasoning.

(a) Nigel is asked to determine whether $(x+7)$ is a factor of $x^3-37x+84$. He substitutes $x=7$ and calculates $7^3-37\times7+84$. This comes to 168, so Nigel concludes that $(x+7)$ is not a factor.

Nigel's conclusion is wrong.

- Explain why Nigel's argument is not valid.
- Show that $(x+7)$ is a factor of $x^3-37x+84$. **[2]**

(b) Sketch the graph of $y=x^3-37x+84$, indicating the coordinates of the points at which the curve crosses the coordinate axes. **[5]**

(c) The graph in part **(b)** is translated by $\begin{pmatrix}1\\0\end{pmatrix}$. Find the equation of the translated graph, giving your answer in the form $y=x^3+ax^2+bx+c$ where a, b and c are integers. **[4]**

8 In this question you must show detailed reasoning.

Show that the only stationary point on the graph of $y = x^2 - 4\sqrt{x}$ is a minimum point at $(1, -3)$. **[7]**

9 In this question you must show detailed reasoning.

A car accelerates from rest along a straight level road. The velocity of the car after 8 s is $25.6\,\text{m s}^{-1}$.

In one model for the motion, the velocity $v\,\text{m s}^{-1}$ at time t seconds is given by $v = 1.2t^2 - kt^3$, where k is a constant and $0 \leqslant t \leqslant 8$.

(a) The model gives the correct velocity of $25.6\,\text{m s}^{-1}$ at time 8 s. Show that $k = 0.1$. **[2]**

A second model for the motion uses constant acceleration.

(b) Find the value of the acceleration which gives the correct velocity of $25.6\,\text{m s}^{-1}$ at time 8 s. **[2]**

(c) Show that these two models give the same value for the displacement in the first 8 s. **[5]**

10 In this question you must show detailed reasoning.

(a) Sketch the gradient function for the curve $y = 24x - 3x^2 - x^3$. **[5]**

(b) Determine the set of values of x for which $24x - 3x^2 - x^3$ is decreasing. **[2]**

11 David puts a block of ice into a cool-box. He wishes to model the mass m kg of the remaining block of ice at time t hours later. He finds that when $t = 5$, $m = 2.1$, and when $t = 50$, $m = 0.21$.

(a) David at first guesses that the mass may be inversely proportional to time. Show that this model fits his measurements. **[3]**

(b) Explain why this model

(i) is not suitable for small values of t, **[1]**

(ii) cannot be used to find the time for the block to melt completely. **[1]**

David instead proposes a linear model $m = at + b$, where a and b are constants.

(c) Find the values of the constants for which the model fits the mass of the block when $t = 5$ and $t = 50$. **[3]**

(d) Interpret these values of a and b. **[2]**

(e) Find the time according to this model for the block of ice to melt completely. **[1]**

END OF QUESTION PAPER

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