Lesson Element – 10.05c Exact trigonometric ratios

Instructions and answers for teachers

These instructions should accompany the OCR resource* 'Lesson Element 10.05c Exact trigonometric ratios' *activity *which supports OCR GCSE (9–1) Mathematics.*

The Activity:

This resource comprises of 2 tasks.

Associated materials:

10.05c Exact trigonometric ratios' Lesson Element learner activity sheet.

Suggested timings:

Task 1: 15-20 minutes

Task 2: 15-20 minutes

ABC – This activity offers an opportunity for English skills development.

123 – This activity offers an opportunity for maths skills development.

Rationale

Learners at GCSE are now expected to know the exact values for sine and cosine of the angles 0°, 30°, 45°, 60° and 90°. Similarly, they are also expected to know the exact values for tangent of the angles 0°, 30°, 45° and 60°.

Finding the exact values using the special triangles 45°, 45° and 90°, and 30°, 60°, and 90° has long been part of the AS Mathematics syllabus. Some AS learners have struggled with this topic, wondering why an exact value is important, so when introducing these trigonometric ratios to GCSE learners a little more support is beneficial.

Assumed Knowledge

These activities assume that learners can:

- Use the sum of the interior angles of a triangle is 180° to find an unknown angle.
- Use the basic properties of isosceles, equilateral and right-angled triangles to find an unknown angle.
- Calculate lengths of sides in right-angled triangles using Pythagoras' Theorem.
- Calculate lengths of sides in right-angled triangles using trigonometric ratios.

Possible Misconceptions

- Learners struggle with the idea of leaving an answer as a fraction and instead want to divide the numerator by the denominator to get a single value, which they then round.

Task 1 – Special Triangles

This task introduces learners to the two special triangles; 45°, 45° and 90°, and 30°, 60° and 90°.

Learners investigate the two triangles through a series of questions and justify the internal angles and length of sides using basic geometric properties and Pythagoras' theorem. This could be done as an initial small group activity or as a whole class discussion, allowing for the recap of properties of triangles and Pythagoras' theorem.

The correct answers are shown below:

1. What type of triangle is this?

Isosceles

2. Explain why two of the angles are 45°.

One angle is 90° so the sum of the other two angles is found by 180 – 90 = 90°.

Isosceles triangles have two equal angles, so 90 ÷ 2 = 45°.

3. What is the length of the remaining side in the triangle?

Using Pythagoras: $x^2 = 1^2 + 1^2$

$x^2 = 2$

$x = \sqrt{2}$

4. Explain why two of the angles in the right-angled triangle are 60° and 30°.

Each angle in an equilateral triangle is 60°.

The angle at the apex of the triangle has been bisected, so its size is 60 ÷ 2 = 30°.

5. What is the height of the triangle?

Using Pythagoras: $h^2 = 2^2 – 1^2$

$h^2 = 3$

$h^2 = \sqrt{3}$

Task 2 – Exact Value Card Sort

Learners use their knowledge of Pythagoras' theorem and trigonometric ratios to express the ratios as exact fractions in surd form. The correct answers are shown below:

sin 30°	cos 30°	tan 30°	sin45°	cos 45°	tan 45°	sin 60°	cos 60°	tan 60°
$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Extension

Encourage learners to look for patterns in their answers.

E.g. sin 30° = cos 60° or sin 60° = cos 30° and sin 45° = cos 45°.

This could lead to a discussion around the complementary nature of these values.

In addition, learners may notice the pattern that was first noted by the Spanish mathematician, Ernesto La Orden. He pointed out the following clever way to remember the exact values of sine and cosine.

Firstly, a table of sine values:

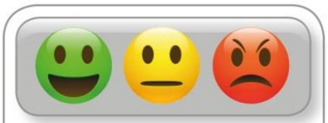
Angle	SINE	
0°	$\frac{\sqrt{0}}{2}$	$= 0$
30°	$\frac{\sqrt{1}}{2}$	$= \frac{1}{2}$
45°	$\frac{\sqrt{2}}{2}$	$= \frac{1}{\sqrt{2}}$
60°	$\frac{\sqrt{3}}{2}$	$= \frac{\sqrt{3}}{2}$
90°	$\frac{\sqrt{4}}{2}$	$= \frac{2}{2} = 1$

You will notice that the numerators in the middle column ascend from 0 to 4.

Now, a table of cosine values:

Angle	COSINE	
0°	$\frac{\sqrt{4}}{2}$	$= \frac{2}{2} = 1$
30°	$\frac{\sqrt{3}}{2}$	$= \frac{\sqrt{3}}{2}$
45°	$\frac{\sqrt{2}}{2}$	$= \frac{1}{\sqrt{2}}$
60°	$\frac{\sqrt{1}}{2}$	$= \frac{1}{2}$
90°	$\frac{\sqrt{0}}{2}$	$= 0$

You will notice that the numerators in the middle column descend from 4 to 0.



We value your feedback

We'd like to know your view on the resources we produce. By clicking on the icon above you will help us to ensure that our resources work for you.

Whether you already offer OCR qualifications, are new to OCR, or are considering switching from your current provider/awarding organisation, you can request more information by completing the Expression of Interest form which can be found here: www.ocr.org.uk/expression-of-interest

Looking for a resource? There is now a quick and easy search tool to help find free resources for your qualification: www.ocr.org.uk/i-want-to/find-resources/