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Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper 4733/01 series overview

The long life of the legacy specification has enabled many centres to become very familiar with the demands of this examination, and many candidates are able to produce good and accurate answers to most of the questions (especially calculations).

This particular paper was in some ways found testing, but there were many good scripts seen. As usual, questions involving calculations were better answered than those requiring interpretation or other verbal answers. Better candidates gave verbal answers that showed that they understood the issues and could apply them, rather than merely regurgitating common phrases. Also as usual, lower ability candidates tended to answer questions that they had seen before (typically, those asked last year) rather than the questions actually asked on the paper in front of them. More specifically, better candidates read questions carefully to ensure that they know *exactly* what they are being asked to do. A typical example where careful reading of the question was required was all three parts of 5(ii).

Conclusions to hypothesis tests are an area that distinguishes better candidates. If the outcome of the test is "reject H_0", the conclusion is that there is significant evidence that H_0 is wrong. If the outcome is "do not reject H_0", the conclusion is that there is insufficient evidence that H_0 is wrong.

Candidates who use the critical region method for hypothesis tests often have more difficulty than those who calculate the probability of a result as extreme as the sample value or more so. In recent sittings of S2, there has been some increase in the number of candidates who seem to be confused between the two methods. Weaker candidates would probably benefit by using the easier method, even though it means an extra technique.

Question 1

1 The results of a random sample of 15 observations of the random variable W can be summarised by

$$n = 15, \qquad \Sigma w = 555, \qquad \Sigma w^2 = 20808.$$

Calculate an unbiased estimate of $\mathrm{E}(W)$ and an unbiased estimate of $\mathrm{Var}(W)$. **[4]**

For most this was a straightforward start to the paper and few incorrect answers were seen. Almost everyone remembered to multiply the variance by $\frac{15}{14}$.

Question 2(i)

2 **(i)** Explain an advantage of using random numbers in sampling. **[1]**

Most candidates could state that the use of random numbers produced an unbiased, or less biased, sample. However, the use of random numbers does not guarantee that the sample is *representative*; it is possible that some subgroups of the population might be over- or under-represented unless further precautions, such as stratification, are taken. The difference between "unbiased" and "representative" is important.

Question 2(ii)

(ii) A random sample of 5 different letters of the alphabet is to be chosen using the following list of random numbers obtained from a calculator.

0.163 0.542 0.007 0.162 0.005 0.022 0.035 0.119 0.188

Determine which letters form the sample, making your method clear. (You do not need to use all the random numbers.) **[2]**

Some candidates struggled to answer this question. Some did not give five letters, as requested, but merely described a method. Some gave the answer to a question that they had met before ("number the population, select using random numbers") rather than this one. Some used the sum of three consecutive digits, and others used the zeroes before the decimal point; both of these methods are very biased (the former method is more likely to produce totals in the middle, such as 13 or 14). Some tried multiplying the digits by the number of letters in the alphabet, many did not seem to know that this number was 26 (24 was often seen). Some used the random numbers in an arbitrary order, which is also not correct. The best candidates made their method clear, as the question asked.

In fact, in order to produce a truly random sample from a population of 26 from numbers in the range 0.000 to 0.999, it is necessary to restrict them, for example to the range 0.000 to 0.987 (as 988 is a multiple of 26). However, for the particular random numbers here the issue was unlikely to arise.

Question 3(i)

3 The number of dust particles in 1 cm^3 of air at a certain location is modelled by the distribution Po(2.7).

(i) Find the probability that, in a region of 1 cm^3 of air at this location, there are at least 5 dust particles. **[2]**

Almost always correct. Only a few gave $1-P(\le 5)$ rather than $1-P(\le 4)$.

Question 3(ii)

(ii) Use a binomial model to calculate the probability that, in 4 adjacent regions of 1 cm^3 of air at this location, there are at least 5 dust particles in exactly 2 of the regions. **[3]**

Almost always correct.

Question 3(iii)

(iii) Use the formula for Poisson probabilities to write down an exact expression for the probability that, in a region of 4 cm^3 of air at this location, there are exactly 12 dust particles, and evaluate your expression. **[2]**

Almost always correct.

Question 4(i)

4 The discrete random variable Y has probability distribution given by

y	0	1	2	3
$P(Y = y)$	0.4	0.2	0.3	0.1

$\bar{Y}$ denotes the mean of 50 random independent observations of Y.

(i) Find the approximate distribution of $\bar{Y}$, giving the value(s) of any parameter(s). **[5]**

Some candidates had forgotten the synoptic S1 technique of finding the expected mean and variance from a discrete distribution. Most knew that the distribution of $\bar{Y}$ would be (approximately) normal, but only the better candidates divided the variance of Y by 50.

Question 4(ii)

(ii) State the possible values taken by $\bar{Y}$ in the range from 1.4 to 1.5 inclusive. **[1]**

Only the best candidates realised that, as Y can take only integer values, $\frac{Y}{50}$ is a discrete distribution with possible values differing by 0.02. (This would be relevant if a calculation involving a continuity correction had been needed.)

Question 5(i)

5 A teacher knows from experience that on average she makes 6 mistakes per session when typing reports. She now adopts a new method of organising her report writing and she finds that in one session she has made 10 mistakes.

(i) Assume first that the number of mistakes she makes can be modelled by a Poisson distribution. Test at the 5% significance level whether the average number of mistakes per session has changed since she adopted the new method. **[7]**

This is a standard question for a significance test for the parameter of a Poisson distribution. Many good answers were seen, but as usual quite a lot of candidates calculated $P(\leq 10)$ or $P(=10)$ instead of $P(\geq 10)$. Those who found the critical region were more likely to make a mistake, particularly if they found the lower tail of the region as well as the upper tail. Less able candidates using this method often did not state the critical region explicitly, thus losing marks.

The conclusion was generally interpreted correctly, with few giving over-assertive answers such as "there is evidence that the number of mistakes has not changed".

Question 5(ii)(a)

(ii) It may be assumed that the teacher makes mistakes randomly and independently of one another.

(a) State another assumption needed if the number of mistakes in a session is to be modelled by a Poisson distribution. **[1]**

In this type of question, candidates are expected to identify any necessary assumption that is not already stated or implied by the given scenario. In this question, there is only one: "mistakes are made at constant average rate". ("Randomly" and "independently" are explicitly stated in the question.)

This condition is widely misunderstood. As usual there were many answers that suggested that mistakes had to be made at an absolutely fixed rate (exactly the same number every hour), and answers to later parts of this question confirmed this serious misunderstanding. Some attempted to use "singly", which as usual is meaningless here; "singly" should not be given as a modelling assumption.

Question 5(ii)(b)

(b) Suggest a reason why this assumption might not hold in practice. **[1]**

The question required a reason why the condition given in part (ii)(a) might not hold, rather than a different condition. The point is that "constant rate" has to hold *throughout a single session*; those who said, for example, "she might make more errors in a session at night than in a session earlier in the day" had missed the point. The most popular answer, which was that anticipated by the examiners, was that the teacher might become tired; another answer to receive credit was that she might improve with practice.

Question 5(ii)(c)

(c) Explain why this means that further information might be needed to calculate the probability that the teacher makes a given number of mistakes in the second hour of a two-hour session. **[1]**

The wording of this question required an expansion of the answer to part (ii)(b). A typical good answer was that you would need to know the mean for the second hour as well as the first.

Question 6(i)

6 The working lifetime T hours of a computer monitor of a certain type is modelled by a continuous random variable with probability density function

$$f(t) = \begin{cases} kt^{-4} & t \geqslant L, \\ 0 & \text{otherwise,} \end{cases}$$

where k and L are constants.

(i) The monitor is sold with a guarantee that its working lifetime is at least T_0 hours (if it fails before time T_0 then it is replaced). State what the model suggests about the value of T_0. **[1]**

Only the best candidates understood what aspect of the model was needed in answering this question. There was numerous misunderstandings of the basic concept of a probability density function; answers such as "*t* [sic] can be very large" were common.

Question 6(ii)

(ii) Show that $k = 3L^3$. **[3]**

A standard request in principle, but the identification and use of the infinite upper limit caused problems for some candidates. Some fudging of the sign was seen.

Question 6(iii)

(iii) Find the variance of T in terms of L. **[6]**

Again this was a standard request and many correct answers were seen. Many realised that they had to find the mean and square it. As usual, those who tried to do the whole thing in one formula, for instance $\int_L^\infty t^2 f(t)\,dt - \left(\int_L^\infty t f(t)\,dt\right)^2$, often made mistakes through cognitive overload. Only a few did not give their answer in terms of L (as opposed to k); it is in fact probably easier to substitute $k = 3L^3$ early on in this part.

Question 6(iv)

(iv) Sketch the graph of the probability density function, and explain whether or not the distribution of T can be well approximated by a normal distribution. **[2]**

The best candidates showed a decreasing graph with a clear truncation to the left, as opposed to an asymptote. Others wrongly indicated an asymptote to the left, or drew part of the curve in the second quadrant. In questions about probability density functions it seems that many lower ability candidates completely ignore the "range" part of the definition.

Most realised that the normal distribution was not appropriate, although some said "it would be valid for large *t*" (this would not be true as the area would be wrong) or thought that the Central Limit Theorem came into it.

Question 7(i)

7 The maximum effective range (MER), X metres, of a long-established brand of wi-fi hub is a continuous random variable modelled by the distribution $N(\mu, \sigma^2)$. In a large random sample it is found that $P(X > 58) = 0.1587$ and $P(X < 40) = 0.3085$.

(i) Calculate the value of μ and the value of σ. **[6]**

This standard question was usually done very well. Only a few did not find *z* values or made sign errors.

Question 7(ii)(a)

(ii) The manufacturers of a new brand of hub claim that its mean MER is at least 10 metres greater than that of the long-established brand. A random sample of 200 values of the MER for this new brand were measured in a variety of environments. The mean MER for this new brand was found to be $(\mu + 8.8)$ metres, where μ is the value found in part **(i)**. A test is to be carried out at the 10% significance level of whether there is evidence that the manufacturer's claim is valid.

(a) State appropriate hypotheses for the test, explaining the meaning of any symbol you use (other than H_0 and H_1). **[3]**

Quite a large number of candidates gave one or both hypotheses in terms of 46, a typical example of "answering a question met before rather than this one".

Surprisingly few seemed to be used to explaining the symbol used. A statement such as "where μ is the mean MER of the new hubs" would be widely considered good practice, but few candidates wrote anything like this. Clearly in this context it is necessary to explain the symbol, so as to avoid confusion with the μ in the question (and on the reformed specification it is expected automatically).

Question 7(ii)(b)

(b) Assuming that the distribution of the MER of the new brand is normal with variance equal to that of the long-established brand, carry out the test. **[5]**

Plenty of better candidates could find their way through this question. The two key aspects were to compare 8.8 with 10 (typically comparing 54.8 with 56), and using a divisor of 200 for the variance.

Those who tried to find a critical region often found the wrong tail, or centred their region on the sample mean (54.8) rather than the hypothesised population mean (56). Weaker candidates often compared 54.8 with 46.

The conclusion proved more challenging. Most who had done the calculations correctly knew that H_0 was to be rejected, but only the best realised that this meant that there was evidence that the manufacturer's claim was false. Some wrote "there is insufficient evidence that the manufacturer's claim was true", but this is not the correct logic of the test.

Question 7(iii)

(iii) Explain whether it was necessary to use the Central Limit Theorem in the test in part **(ii)**. **[1]**

This type of question was better answered than in some previous years, with better candidates knowing that they had been told to assume that the parent distribution was normal so the Central Limit Theorem was not required. The size of the sample does not come into it, nor does the variance. (The Central Limit Theorem never has anything to do with the variance.)

Question 8(i)

8 *In this question you should justify the use of any approximate distributions.*

The discrete random variable M has the distribution $B(60, p)$. A test of the hypotheses $H_0: p = 0.04$, $H_1: p > 0.04$ is carried out at a significance level as close to 5% as possible.

(i) Use an appropriate approximation to determine the critical region for the test. You should show the values of any relevant probabilities. **[7]**

In principle this question was well answered, though only the best candidates not only justified the use of the Poisson approximation with the two correct criteria but also showed both of the probabilities that straddled 0.05. Less good candidates did not state the critical region clearly, perhaps because they could not remember whether it was the rejection region or the acceptance region. A good answer was "$P(\geq 5) = 0.0959$, $P(\geq 6) = 0.0357$, so the critical region is ≥ 6". Only a few tried to use a normal approximation.

Question 8(ii)

(ii) Using the critical region found in part **(i)**, the hypothesis test is carried out 100 times. Each time the value of p is 0.04. State the expected number of times that the test will result in a Type I error. **[1]**

Many did the correct method here (though there is no need to round off 3.57 to 4). Some worked out 100×0.04 instead of 100×0.0357.

Question 8(iii)

(iii) Use an appropriate approximation to find the probability of a Type II error when $p = 0.15$. **[7]**

Most candidates realised that different approximations were needed in parts (i) and (iii), and some excellent answers were seen. If numerical criteria are used to justify the approximation, the relevant values of *np* and *nq* need to be stated explicitly, so that "9 > 5, 51 > 5" is required.

Many could apply the concept of Type II error correctly to give a final probability less than 0.5, and the continuity correction was very often correct. Some did not realise that they had to use the critical region from part (i), instead attempting to find a new critical region from the new distribution.

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