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Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper 4723/01 series overview

The Assessment Specialists marking this paper were delighted to deal with many excellent scripts where due care had been taken with the presentation and where the necessary detail was easy to follow and assess. Over 4% of the candidates recorded a total of 70 or more out of the 72 available marks and over 25% recorded a total of 60 or more. The paper proved accessible to the vast majority even though it included a few challenging requests. There were very few candidates who made little or no progress with the questions; only 1% of the candidates recorded a total of fewer than 10 marks out of 72.

Candidates appeared to have had sufficient time to complete the paper. Given that there was no need to rush to complete the paper, it is disappointing to note that the general standard of presentation was not as good as it has been in previous sessions. Writing that was sometimes difficult to decipher, solutions wandering haphazardly all over the page, solutions restarting but with no signal that this was happening were features that were often noted. Candidates do themselves no favours when they fail to appreciate that effective communication is a vital aspect of writing examinations.

Many scripts did reveal uncertainty about one particular algebraic technique – managing fractions and rational functions correctly. Correct work in Q.3(ii) often led to the equation $y = 6\left(x - \frac{1+a}{2}\right)$. On this occasion, this form of the equation earned the accuracy mark but it was noticeable how often candidates developed this and had $y = 6\left(x - \frac{1}{2} + \frac{a}{2}\right)$ as the next step; candidates did not appreciate the implied existence of brackets. At the start of Q.7, it was surprising how often $3\tan 2x = \frac{6\tan x}{3 - 3\tan^2 x}$ was seen. At the start of the solution of Q.9(ii), candidates had to rearrange $y = \frac{4}{2x+1}$ to express x in terms of y and a significant number of candidates was unable to do this correctly. Many reached $x = \frac{\frac{4}{y} - 1}{2}$ and then struggled. Some candidates, perhaps unsure of their own ability at dealing with this, chose to square this to find an expression for x^2. In other cases, attempts at simplification led to the appearance of the term $\frac{8}{y}$.

Question 1

1 Use Simpson's rule with four strips to find an approximation to

$$\int_1^5 e^{\frac{2}{x}}\,dx.$$ **[3]**

This opening question was answered very well and most candidates had no difficulty in reaching a correct answer. There were a few instances of candidates using an incorrect sequence of coefficients and a few others where the correct expression involving e was produced but was followed by an incorrect answer.

Question 2

2 Solve the inequality $|4x+3| < |x-8|$, showing all your working. **[5]**

Candidates opting to proceed by squaring both sides of the inequality tended to be slightly more successful than those considering linear equations or inequalities. The former group handled the quadratic equation or inequality well, usually taking the helpful step of dividing through by 5 before using factorisation, or sometimes the quadratic formula, to find the two critical values.

Those using two linear equations to find the critical values often produced very good graphs of $y = |4x+3|$ and $y = |x-8|$ and used these to note the two linear equations that needed to be solved. But, in other cases, the process seemed more haphazard and sign slips were common. There were some instances of as many as four linear equations being considered and these did not always produce just two values of *x*.

Care was needed in the way the final answer was presented and this was not always evident. The appearance of $\leq$ signs meant that the final mark was not available but other attempts revealed some uncertainty about inequalities; answers such as $x > -\frac{11}{3}$, $x < 1$ did not earn full credit. The expected answer was $-\frac{11}{3} < x < 1$ or a clear equivalent.

Question 3(i)

3 A curve has equation $y = 3\ln(2x - a)$, where a is a positive constant. The curve crosses the x-axis at the point P.

(i) Sketch the curve and determine the x-coordinate of P in terms of a. **[3]**

Most candidates produced an acceptable graph in part (i) although some graphs existed only in the first quadrant. Showing the asymptote and giving its equation were not required on this occasion, although it is good practice and some candidates meticulously did so. Finding the *x*-coordinate of the point *P* caused problems, apparently due in many cases to the presence of the positive constant *a*. Often candidates correctly reached the equation $2x - a = 1$ but then seemed unsure how to proceed. Other candidates decided that the location of *P* required $2x - a = 0$, showing a lack of awareness of the nature of the logarithm involved. A different approach adopted quite commonly was to consider the transformations required to transform the graph of $y = \ln x$ to the graph of $y = 3\ln(2x - a)$. This approach was seldom successful; details of the transformations required were not correct and the incorrect answer $x = \frac{1}{2}a + 1$ was frequently seen.

Question 3(ii)

(ii) Find an equation of the tangent to the curve at P. **[3]**

In part (ii), most candidates differentiated correctly but proceeding to find the equation of the tangent proved more challenging. Many used a general expression for the gradient in their equation and offered answers such as $y = \frac{6}{2x - a}\left(x - \frac{a+1}{2}\right)$. Whether the presence of *a* was responsible for this common error was not clear but only approximately 40% of the candidates earned all three marks in part (ii).

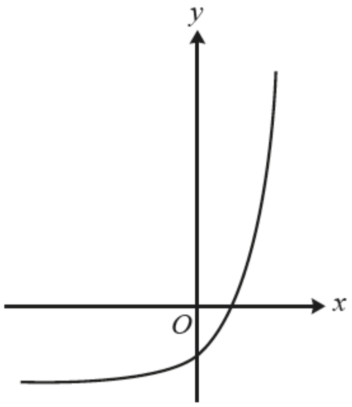
Question 4

4 A curve has equation $y = \frac{2x^2 + 1}{x^4 + 30}$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and hence determine the exact coordinates of the stationary points on the curve. **[7]**

There was generally excellent use of the quotient rule and most candidates earned the first two marks. Care and accuracy were then needed if a successful outcome was to follow. Candidates who avoided sign slips in the simplification were faced with the equation $-4x^5 - 4x^3 + 120x = 0$ and some could make no progress at all with this. Others promptly cancelled *x* from the equation and therefore usually lost one of the stationary points. Many, having dealt with the common factor *x*, did recognise a disguised quadratic equation but further errors often occurred. It was common for $x^2 = 5$ to lead to only the value $x = \sqrt{5}$. It was pleasing to note the number of fully correct, clear solutions but a significant number of the candidates did miss one or two of the three correct stationary points. The question asked for exact coordinates and so decimal equivalents, possibly obtained using an equation solver routine on a calculator, could not earn the relevant credit.

Question 5(i)

5



The diagram shows the curve $y = \mathrm{f}(x)$, where f is the function defined for all real values of x by $\mathrm{f}(x) = \mathrm{e}^{2x} - 3$.

(i) State the range of f. **[1]**

There were many cases of $\mathrm{f}(x) \geq -3$ being offered as the answer for part (i); this suggests some uncertainty about the nature of an exponential function and a failure to recognise the existence of an asymptote parallel to the *x*-axis. There were also cases of the answer involving 3 rather than –3 and raises questions about an appreciation of the term range, especially as the diagram shows the relevant curve.

Question 5(ii)

(ii) Find an expression for $\mathrm{f}^{-1}(x)$. **[2]**

This part was answered well and the vast majority of the candidates earned both marks. A few gave the answer in terms of *y* and there were some instances of the answer being given as $\frac{1}{2}\ln x + 3$.

Question 5(iii)

(iii) The curve $y = \mathrm{e}^{x}$ can be transformed to the curve $y = \mathrm{f}^{-1}(x)$ by means of a stretch, a translation and a reflection in that order. Give details of these three transformations. **[3]**

By contrast to the previous parts, part (iii) was a more demanding request and only 21% of the candidates succeeded in earning all three marks. Most candidates correctly mentioned reflection in the line $y = x$ but details of the other two transformations were often wrong and it seemed that candidates had not paid sufficient attention to the order in which the three transformations were to be applied. A translation of –3 units parallel to the *x*-axis and a stretch with scale factor $\frac{1}{2}$ parallel to the *y*-axis were common errors.

Question 5(iv)

(iv) Sketch the curve $y = |f(x)|$. Given that the equation $|f(x)| = k$ has two distinct roots, determine the set of possible values of the constant k. **[3]**

Most candidates drew an acceptable sketch of $y = |f(x)|$ and the sketch enabled many alert candidates to realise immediately that the set of possible values of the constant k was $0 < k < 3$. Some candidates used $\le$ signs or had one of the end points wrong but partial credit was available in such cases. For a large number of candidates, the requirement for the equation to have two distinct roots led them to an algebraic approach, usually based on $b^2 - 4ac > 0$; this approach seldom led to any success.

Question 6(a)

6 **(a)** A reservoir is being filled with water at a constant rate of 15 cubic metres per minute. At the instant when the depth of the water is x metres, the volume of water in the reservoir is V cubic metres where

$$V = 2(5+2x)^3 - 250.$$

Find the rate at which the depth of the water is increasing at the instant when $x = 1.6$. **[4]**

This part was generally answered well with 65% of the candidates recording full marks. It was pleasing to see many attempts with the working set out clearly and showing clear understanding of how the three differential coefficients are related. The attempt to find $\frac{dV}{dx}$ was usually correct and any errors occurred subsequently. Some merely substituted 1.6 into this derivative and claimed that as their answer. Other candidates did involve the value 15 but carried out the wrong calculation.

Question 6(b)(i)

(b) In an experiment, the mass of a substance is increasing exponentially. At a time t hours after the start of the experiment, the mass, m grams, of the substance is given by

$$m = Ae^{\lambda t},$$

where A and λ are constants. It is given that, at the instant when $t = 15$, the mass is 48 grams and the rate at which the mass is increasing is 1.2 grams per hour.

(i) Find the values of A and λ. **[4]**

Just over half of the candidates answered part (b)(i) without trouble. They substituted the appropriate values and produced the two equations $Ae^{15\lambda} = 48$ and $A\lambda e^{15\lambda} = 1.2$. Most then realised that the value of λ could be found easily. For other candidates, there was a variety of problems. Some, having obtained these two correct equations, could not see how to proceed and solutions tended to meander, sometimes involving logarithms, and seldom with any success. There were errors with the differentiation and $\frac{dm}{dt} = \lambda t A e^{\lambda t}$ was a common mistake; some candidates attempted to proceed with this or the correct derivative without substituting $t = 15$. A more fundamental mistake made by a significant number of candidates was to assume that the rate of increase of the mass was constant; this betrays a lack of understanding of the nature of exponential increase. This basic mistake led to candidates claiming the value of A as 30 or to attempting, a solution using a value of 49.2 for m when t is 16.

Question 6(b)(ii)

(ii) Find the value of t for which the mass is 70 grams. **[2]**

The vast majority of candidates knew how to proceed with part (b)(ii) although earlier errors prevented a successful outcome in some cases.

Question 7(i)

7 It is given that there is exactly one value of x, where $0 < x < \pi$, that satisfies the equation

$$3\tan 2x - 8\tan x = 4.$$

(i) Show that $t = \sqrt[3]{\frac{1}{2} + \frac{1}{4}t - \frac{1}{2}t^2}$, where $t = \tan x$. **[3]**

There was a mixed response to the different parts of this question with parts (i) and (iii) generally being answered well but with the other two parts causing problems for some candidates. Candidates knew the identity for $\tan 2x$ and were generally able to manipulate the equation accurately to confirm the given equation. There were some sign errors on the way and some candidates concluded with a square root sign rather than a cube root. With the answer given in the question, it is expected that candidates will be careful in providing a clear and accurate solution.

Question 7(ii)

(ii) Show by calculation that the value of t satisfying the equation in part **(i)** lies between 0.7 and 0.8. **[2]**

There was considerable uncertainty apparent in many attempts at part (ii). Often candidates substituted 0.7 and 0.8 into the expression $\sqrt[3]{\frac{1}{2} + \frac{1}{4}t - \frac{1}{2}t^2}$ and were surprised when an expected sign change was not apparent. Some did then belatedly subtract 0.7 and 0.8 from the values and were able to claim a sign change. Those setting out to calculate values of $\sqrt[3]{\frac{1}{2} + \frac{1}{4}t - \frac{1}{2}t^2} - t$ were more successful although they were required to draw attention to the change in sign to earn the second mark. Some candidates rearranged the equation in t before substituting; this was fine provided candidates made it clear what their new equation was. There were several instances of candidates failing to communicate and mysterious values appeared as if from nowhere; no marks can be given in such circumstances. Some candidates were confused between values of t and values of x and tried to proceed by substituting $\tan 0.7$ and $\tan 0.8$.

Question 7(iii)

(iii) Use an iterative process based on the equation in part **(i)** to find the value of t correct to 4 significant figures. Use a starting value of 0.75 and show the result of each iteration. **[3]**

This confusion between *t* and *x* also marred a few attempts at part (iii) as candidates used a starting value of $\tan 0.75$ instead of 0.75 but generally this part was answered well and 74% of candidates duly earned all three marks. There were a few cases where candidates concluded with the value 0.7427 or with 0.743. This sort of iterative process always appears in some form in this unit and it is surprising when candidates are not able to reach a correct answer.

Question 7(iv)

(iv) Solve the equation $3\tan 4y - 8\tan 2y = 4$ for $0 < y < \frac{1}{2}\pi$. **[2]**

This part was more challenging with many candidates not recognising how this request was related to the earlier parts. Some tried to solve the equation without any reference to earlier parts and others decided that the value of *y* would be half their answer from part (iii). However, it was pleasing to note the clarity of thought displayed by many candidates who recognised that they needed to solve $\tan 2y = 0.7428$ and who managed to do so efficiently.

Question 8(a)

8 (a) Given that α satisfies the equation

$$3\sin(\alpha + 60°) - 3\cos(\alpha + 30°) = \operatorname{cosec}^2\alpha,$$

find the exact value of $\sin\alpha$. **[4]**

This part was generally answered well and 60% of the candidates earned all four marks. The necessary expansions were usually correct although there were some uncertainties regarding signs and brackets. A number of candidates seemed to lack the presentational skills that enabled them to be meticulous about presenting mathematical expressions accurately; in a few cases, the left-hand side of the equation reduced to zero. Most candidates dealt correctly with the right-hand side of the equation and duly reached $\sin^3\alpha = \frac{1}{3}$. A correct conclusion usually followed but there were several instances of a final value for $\sin\alpha$ of $\sqrt{\frac{1}{3}}$ or $\pm\sqrt[3]{\frac{1}{3}}$.

Question 8(b)

(b) It is given that β satisfies the equation

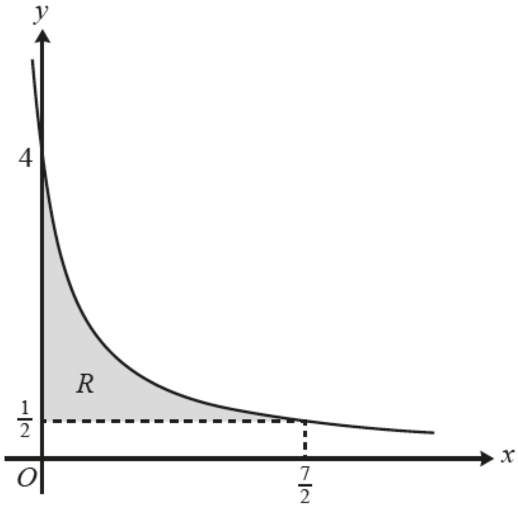
$$\sin 4\beta \sec^3 \beta = 8 \sin \beta + 2.$$

By first expressing $\sin 4\beta$ in terms of $\sin 2\beta$ and $\cos 2\beta$, find the exact value of $\sin \beta$. **[7]**

This was a more challenging request but most candidates were able to earn at least some of the marks. Most duly expressed $\sin 4\beta$ as $2\sin 2\beta \cos 2\beta$ and then were able to convert the equation to one involving $\sin\beta$ and $\cos\beta$. Care and precision were then needed in simplifying the equation and these qualities were missing in many cases as candidates produced lengthy and involved attempts. In many cases, candidates would have benefited by a moment's consideration of each step of their solutions. For example, many solutions reached the correct equation $8\sin\beta - \dfrac{4\sin\beta}{\cos^2\beta} = 8\sin\beta + 2$ but the obvious next step of $-\dfrac{4\sin\beta}{\cos^2\beta} = 2$ was not taken. It was pleasing to note the number of candidates who were able to reach the correct quadratic equation in $\sin\beta$ with accurate and efficient work. A final decision concerned the value of $\sin\beta$ and not all of the candidates reaching this stage realised that only one of the two roots of the equation was acceptable as the value of $\sin\beta$.

Question 9(i)

9



The diagram shows part of the curve $y = \frac{4}{2x+1}$. The shaded region R is enclosed by the curve and the lines $x = 0$ and $y = \frac{1}{2}$.

(i) Find the exact area of R, giving your answer in the form $a\ln 2 + b$ where a and b are constants. **[4]**

There were two approaches taken for part (i) and both methods were successful in many cases; 52% of the candidates earned all four marks. The majority of candidates attempted the area under the curve and subtracted the area of the rectangle. Sometimes the integral appeared as $4\ln(2x+1)$ and there were a few cases of the use of incorrect limits. A considerable number of candidates ignored the request to give the area in terms of $\ln 2$ and offered $2\ln 8 - \frac{7}{4}$ as their answer. The second approach involved finding the area between the curve and the *y*-axis and this was often carried out correctly. Candidates using this method had slightly different steps to take, expressing *x* in terms of *y* correctly and having a little more to do to reach an answer in terms of $\ln 2$.

Question 9(ii)

(ii) The region R is rotated completely about the ***y*-axis**. Find the exact volume of the solid produced, giving your answer in the form $c\ln 2 + d$ where c and d are constants. **[7]**

This part formed a suitable final request in the paper. There were candidates who attempted to rotate about the *x*-axis or who made little sensible progress in their attempts to rotate about the *y*-axis. Most candidates were able to earn at least some of the marks but the various techniques needed did test the abilities of many candidates. There were many errors in attempting to express x^2 in terms of *y*, in integrating terms involving negative powers, in converting expressions such as $-2\ln 4 + 2\ln\frac{1}{2}$ to a form involving $\ln 2$; and not all remembered to include π in their answer. A suspicion that not all candidates fully appreciated the process of finding the volume arose when, after going through the whole process successfully, a number of candidates then proceeded to attempt to subtract the volume of a cylinder. Nevertheless, it is pleasing to record that 22% of the candidates did deal with all these aspects successfully and concluded by earning all seven marks.

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