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Examiners' report

MATHEMATICS B (MEI)

H640
For first teaching in 2017

H640/03 Summer 2018 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper H640/03 series overview

This was the third and final paper for this new A Level and all the candidates had prepared for this examination in one year. The marks were generally very good as many candidates are also Further Mathematics candidates. This paper contributes 27.3% of the total A-level and assesses content solely from pure mathematics.

To do well in this component, candidates need to be able to apply their knowledge of the syllabus content in a variety of modelling contexts and to make efficient use of calculator technology.

Section A overview

This section contained questions on pure maths of a range of length and difficulty.

Question 1

1 Triangle ABC is shown in Fig. 1.

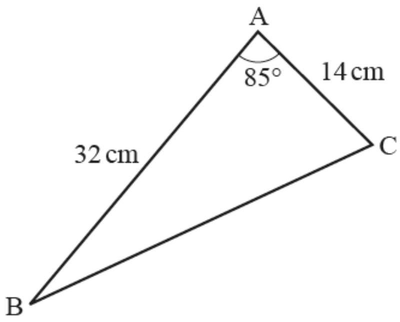


Fig. 1

Find the perimeter of triangle ABC. **[3]**

For the majority of candidates this provided a straightforward start to the paper. The few candidates who did not score full marks either misread the question and only found the length of *BC*, did not give sufficient accuracy in their answer. A small minority of candidates did not recall accurately the Cosine Rule.

AfL

This question is in degrees, but many questions at A Level involve the use of radians. It is important that candidates are confident switching between units on their calculators. The specification advice to explicitly write down any expressions to be evaluated by calculator would ensure partial credit where the incorrect setting on the calculator is used.

Question 2

2 The curve $y = x^3 - 2x$ is translated by the vector $\begin{pmatrix} 1 \\ -4 \end{pmatrix}$. Write down the equation of the translated curve. **[2]**

Those candidates who knew to substitute *x*-1 generally scored fully here.

AfL

Note that the question did not require the answer to be expanded and some candidates wasted time doing this. Whilst this is an important skill, if required the question will specify a specific format for the answer.

Question 3

3 Fig. 3 shows a circle with centre O and radius 1 unit. Points A and B lie on the circle with angle AOB = θ radians. C lies on AO, and BC is perpendicular to AO.

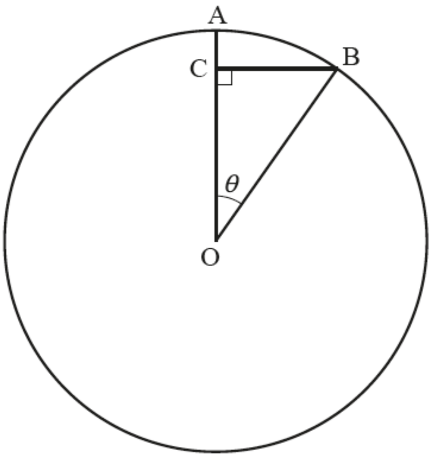
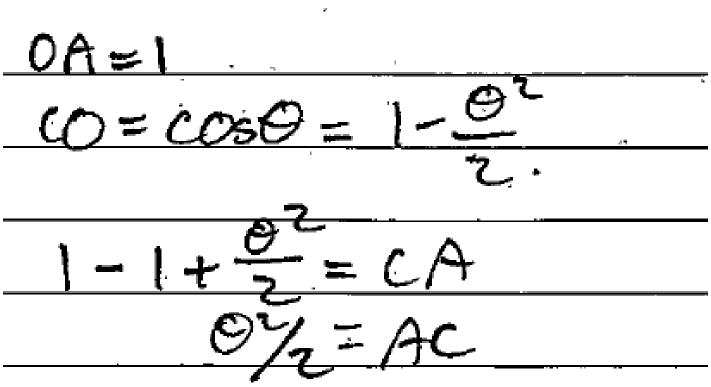


Fig. 3

Show that, when θ is small, $AC \approx \frac{1}{2}\theta^2$. **[2]**

This is the first example of a 'Show that' question in this paper and candidates could not score unless they explained the logic of their initial expression

Exemplar 1



$OA = 1$

$CO = \cos\theta = 1 - \frac{\theta^2}{2}$.

$1 - 1 + \frac{\theta^2}{2} = CA$

$\theta^2/2 = AC$

The candidate knows the small angle approximation to use but does not explain where their first equation for *CA* comes from. Therefore no marks can be earned.

Question 4 (i)

4 In this question you must show detailed reasoning.

A curve has equation $y = x - 5 + \frac{1}{x-2}$. The curve is shown in Fig. 4.

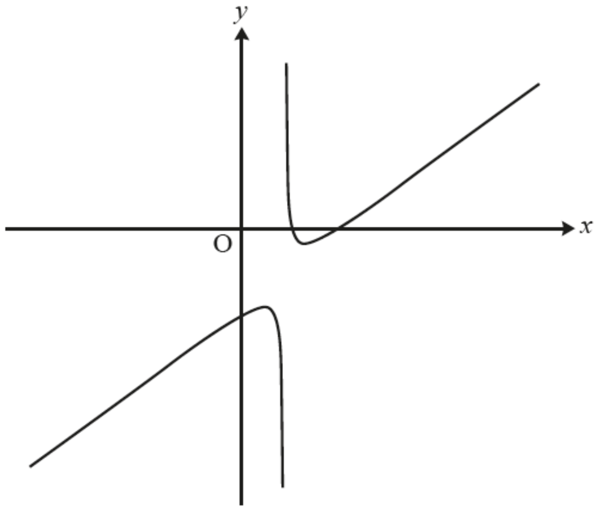


Fig. 4

(i) Determine the coordinates of the stationary points on the curve. **[5]**

Most candidates were able to score full marks here following correct differentiation and solution of what ended up as a quadratic equation. Many different ways were used to solve the equation usually without any wrong working. Examiners were pleased to see correct notation used in this question.

Question 4 (ii)

(ii) Determine the nature of each stationary point. **[3]**

Again most candidates were successful in classifying the stationary points with use of the second derivative being the most common method. A few considered the gradient either side of each turning point and then reasoned their way to a correct conclusion.

Question 4 (iii)

(iii) Write down the equation of the vertical asymptote. **[1]**

There appeared to be confusion as to the meaning of vertical asymptote which led to a low success rate for this part.

Question 4 (iv)

(iv) Deduce the set of values of x for which the curve is concave upwards. **[1]**

As in part (iii), a large proportion of the candidates struggled with this part.

AfL

The OCR B (MEI) H640 specification defines the terms "concave upwards" and "concave downwards" as those that will be used in examination questions.

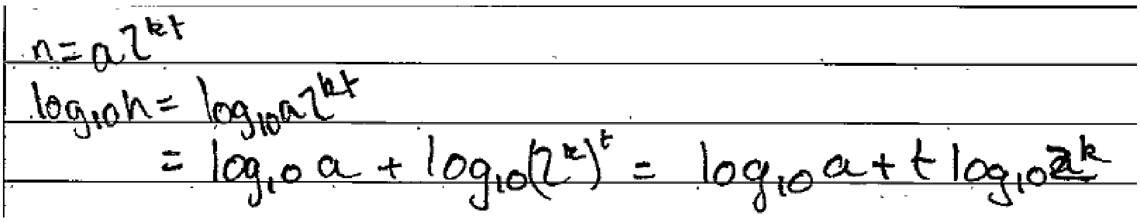
Question 5 (i)

5 A social media website launched on 1 January 2017. The owners of the website report the number of users the site has at the start of each month. They believe that the relationship between the number of users, n, and the number of months after launch, t, can be modelled by $n = a \times 2^{kt}$ where a and k are constants.

(i) Show that, according to the model, the graph of $\log_{10} n$ against t is a straight line. **[2]**

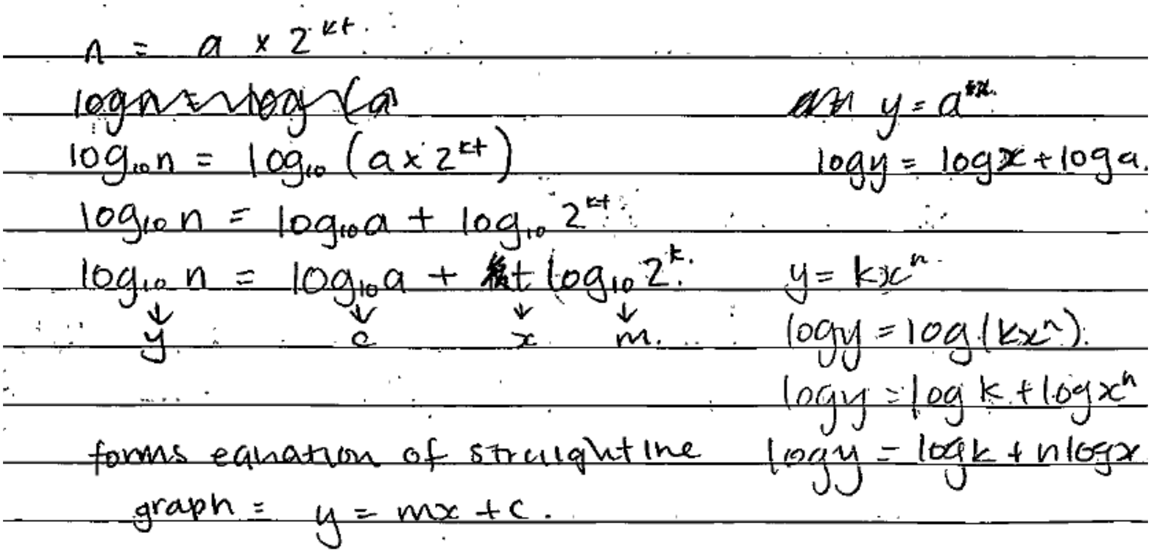
Most candidates could manage the use of logs and rearranging but quite a number did not finish off and explain why it was a straight line.

Exemplar 2



The candidate correctly rearranges but does not relate their equation to $y = mx + c$ so only scores 1.

Exemplar 3


$n = a \times 2^{kt}$

~~log n = log (a~~

$\log_{10} n = \log_{10} (a \times 2^{kt})$

$\log_{10} n = \log_{10} a + \log_{10} 2^{kt}$

$\log_{10} n = \log_{10} a + t \log_{10} 2^{k}$

y ↓ c ↓ x ↓ m ↓

forms equation of straight line graph = $y = mx + c$.

an $y = a^{kx}$

$\log y = \log x + \log a$.

$y = kx^{n}$

$\log y = \log (kx^{n})$

$\log y = \log k + \log x^{n}$

$\log y = \log k + n \log x$

The candidate makes doubly sure of the final mark by saying it is $y = mx + c$ and showing which parts of their equation correspond to *m, c* etc.

Question 5 (ii)

(ii) Fig. 5 shows a plot of the values of t and $\log_{10} n$ for the first seven months. The point at $t = 1$ is for 1 February 2017, and so on.

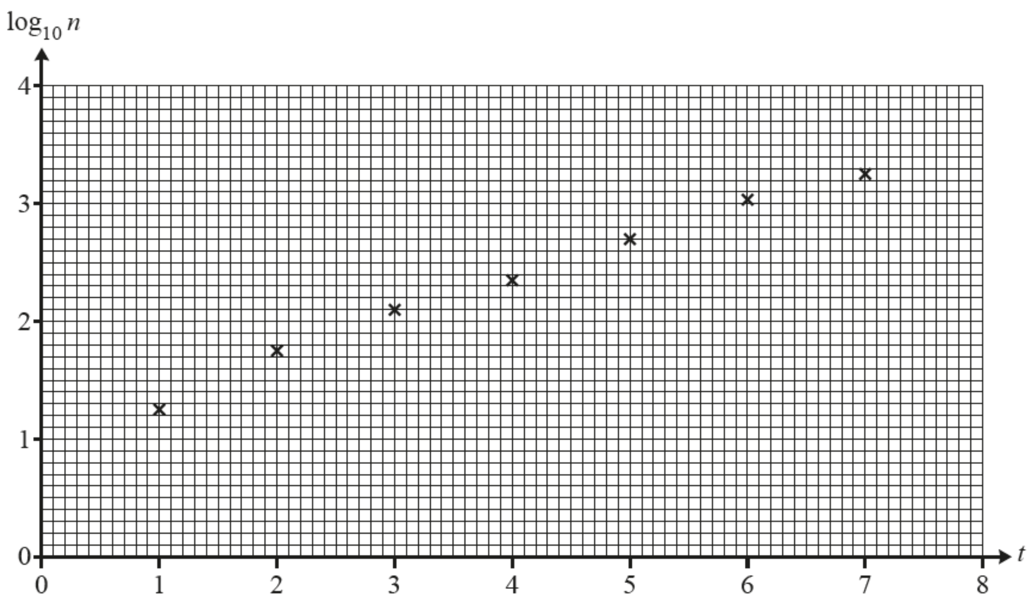


Fig. 5

Find estimates of the values of a and k. **[4]**

Examiners were surprised how few actually drew in a line of best fit. There is the possibility that candidates drew their line of best fit on the question paper, rather than the answer booklet. Candidates should be reminded that it is only the answer booklet that is seen by the examiner, therefore any rough work, sketches, diagrams or annotations that are drawn on the question paper will not gain credit.

AfL

Choosing two of the points to determine the gradient is not generally the best method but finding the gradient of their line is. It is possible that a candidate may have determined that all the points were generally close to the 'line'. This assumption was not penalised in this assessment.

Question 5 (iii)

(iii) The owners of the website wanted to know the date on which they would report that the website had half a million users. Use the model to estimate this date. **[4]**

Many started well but many candidates did not correctly translate their *t* into the correct date or round up.

AfL

One of the features of the reformed qualification criteria is the increased emphasis on modelling. Candidates should ensure that their final answer reflects the context of the original problem and not only focus on the mathematical techniques.

Question 5 (iv)

(iv) Give a reason why the model may not be appropriate for large values of t. **[1]**

A suitable reason was generally supplied for this part, even if the previous part was not correct.

AfL

Candidates should be reminded that subsequent questions related to the model may not require a successful answer to previous parts using the model. Good exam practice involves reading the full question before answering the individual parts, especially if the first parts appear challenging.

Question 6

6 Find the constant term in the expansion of $\left(x^2 + \frac{1}{x}\right)^{15}$. **[2]**

This was a standard question for which many fully correct solutions were provided. Whilst not required, a clear justification of why ${}^{15}C_5$ is needed may help ensure the correct answer of 3003 is obtained, and could gain partial credit if a minor error is made.

Question 7

7 **In this question you must show detailed reasoning.**

Fig. 7 shows the curve $y = 5x - x^2$.

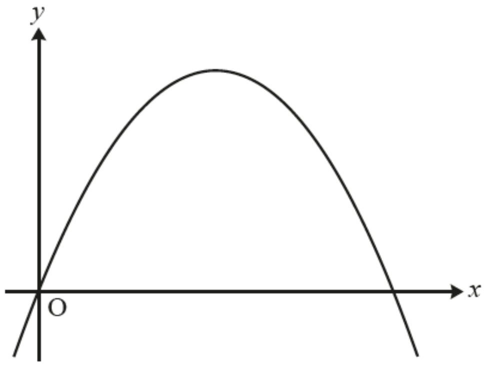


Fig. 7

The line $y = 4 - kx$ crosses the curve $y = 5x - x^2$ on the x-axis and at one other point.

Determine the coordinates of this other point. **[8]**

This was a question where candidates generally understood the necessary steps required. Some solutions lost a mark because they did not show why $x = 0$ should be rejected in favour of $x = 5$ being used. Also a few candidates got a little bogged down by focussing on the discriminant.

Question 8 (i)

8 A curve has parametric equations $x = \dfrac{t}{1+t^3}$, $y = \dfrac{t^2}{1+t^3}$, where $t \neq -1$.

(i) **In this question you must show detailed reasoning.**

Determine the gradient of the curve at the point where $t = 1$. **[5]**

This part saw many attempts at the correct process, but there were some mistakes seen in applying the quotient rule generally with confusion over signs. More difficulty was experienced by those who chose to use the product rule but did not appreciate the need to apply the chain rule to the differentiation of $(1+t^3)^{-1}$.

AfL

Possibly the use of the product rule to prove the quotient rule leads some candidates to assume that that is the method to use to differentiate quotients. Candidates should know the difference and be able to apply both the product rule and the quotient rule accurately as appropriate.

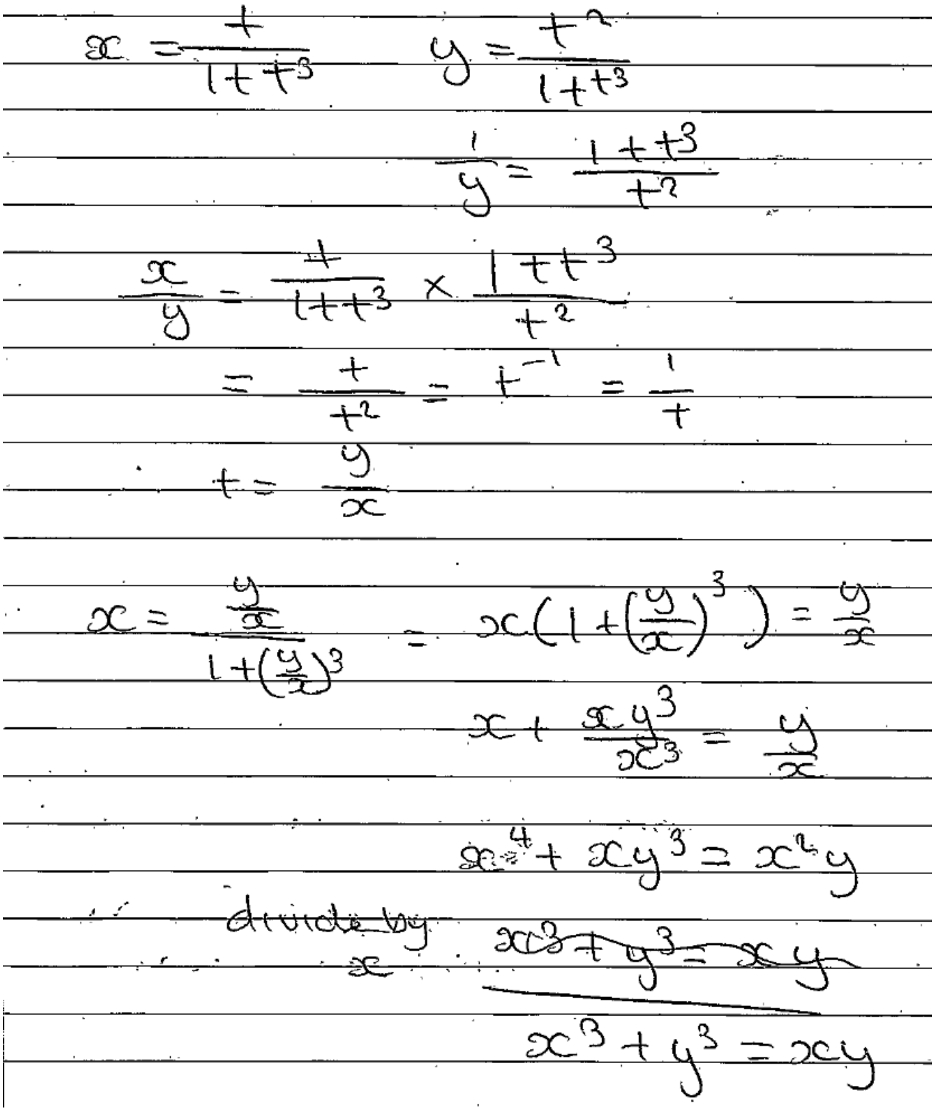
Question 8 (ii)

(ii) Verify that the cartesian equation of the curve is $x^3 + y^3 = xy$. **[3]**

This part was generally started well, but many candidates struggled to complete the argument to score fully.

The majority tackled this question by starting with finding an expression for $x^3 + y^3$ in terms of *t*, but a few chose to use the expression for *x* or *y* and make *t* the subject then continue to eliminate *t* from the expressions and generally complete the argument satisfactorily.

Exemplar 4


$x = \frac{t}{1+t^3}$ $y = \frac{t^2}{1+t^3}$

$\frac{1}{y} = \frac{1+t^3}{t^2}$

$\frac{x}{y} = \frac{t}{1+t^3} \times \frac{1+t^3}{t^2}$

$= \frac{t}{t^2} = t^{-1} = \frac{1}{t}$

$t = \frac{y}{x}$

$x = \frac{\frac{y}{x}}{1+(\frac{y}{x})^3} = x(1+(\frac{y}{x})^3) = \frac{y}{x}$

$x + \frac{xy^3}{x^3} = \frac{y}{x}$

$x^4 + xy^3 = x^2y$

divide by x ~~$x^3 + y^3 = xy$~~

$x^3 + y^3 = xy$

This candidate does full proof of the result gaining all 3 marks.

Exemplar 5

$$\left(\frac{t}{1+t^3}\right)^3 + \left(\frac{t^2}{1+t^3}\right)^3 = \frac{t^3}{(1+t^3)^3} + \frac{t^6}{(1+t^3)^3}$$

$$= \frac{t^6+t^3}{(1+t^3)^3} = \frac{t^3(t^3+1)}{(1+t^3)^3} = \frac{t^3}{(1+t^3)^2} = xy$$

Because $xy = \left(\frac{t}{1+t^3}\right)\left(\frac{t^2}{1+t^3}\right) = \frac{t^3}{(1+t^3)^2}$

$$\frac{dx}{dt} = (1+t^3)^{-1} - 3t^3(1+t^3)^{-2}$$

$$\frac{dy}{dt} = 2t(1+t^3)^{-1} - 3t^4(1+t^3)^{-2}$$

When $t=1$, $\frac{dx}{dt} = -\frac{1}{4}$

$$\frac{dy}{dt} = \frac{1}{4}$$

$$\frac{dy}{dx} = \frac{1}{4} \div -\frac{1}{4} = -1$$

This candidate shows appropriate initial working to verify the equation, but misses out on the final mark, as they do not state what they have shown.

Question 9 (i)

9 The function $f(x) = \frac{e^x}{1-e^x}$ is defined on the domain $x \in \mathbb{R}$, $x \neq 0$.

(i) Find $f^{-1}(x)$. **[3]**

Most candidates were well prepared for this standard question. Errors seen tended to be in forgetting to interchange *x* and *y*.

Question 9 (ii)

(ii) Write down the range of $f^{-1}(x)$. **[1]**

The majority of candidates correctly identified that the range of $f^{-1}(x) \neq 0$

Question 10 (i)

10 Point A has position vector $\begin{pmatrix} a \\ b \\ 0 \end{pmatrix}$ where a and b can vary, point B has position vector $\begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$ and point C has position vector $\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$. ABC is an isosceles triangle with AC = AB.

(i) Show that $a - b + 1 = 0$. **[4]**

This part was generally answered well.

Question 10 (ii)

(ii) Determine the position vector of A such that triangle ABC has minimum area. **[6]**

This non-standard problem led to many different approaches but finding the midpoint of BC seemed the most logical start. The need to find a relevant vector such as A to the midpoint of *BC* in terms of one variable was appreciated by most candidates. The challenging part of this question was to find *a* for the minimum area and then use it to find the correct vector.

Section B overview

This section consisted of questions testing comprehension of an article on arithmetic and geometric means and their different properties. It is recommended that candidates read the full article before attempting to answer any of the questions.

Question 11 (i)

11 Line 8 states that $\frac{a+b}{2} \geqslant \sqrt{ab}$ for $a,\ b \geqslant 0$. Explain why the result cannot be extended to apply in each of the following cases.

(i) One of the numbers a and b is positive and the other is negative. **[1]**

This was answered well by the majority of candidates.

Question 11 (ii)

(ii) Both numbers a and b are negative. **[1]**

This was also answered well.

Question 12

12 Lines 5 and 6 outline the stages in a proof that $\frac{a+b}{2} \geqslant \sqrt{ab}$. Starting from $(a-b)^2 \geqslant 0$, give a detailed proof of the inequality of arithmetic and geometric means. **[3]**

Many fully correct proofs seen here. However some candidates stated that $a^2 + b^2 = (a + b)^2$.

Question 13

13 Consider a geometric sequence in which all the terms are positive real numbers. Show that, for any three consecutive terms of this sequence, the middle one is the geometric mean of the other two. **[3]**

Most candidates were able to give general expressions for three consecutive terms of a geometric series and their response progressed successfully. However, some candidates lost a mark by not correctly completing their explanation by relating to the middle term.

Question 14 (i)

14 **(i)** In Fig. C1.3, angle CBD $= \theta$. Show that angle CDA is also θ, as given in line 23. **[2]**

This question drew on prior knowledge from GCSE. Only few managed to give both 'angles in a triangle' and 'angle in a semicircle'.

Question 14 (ii)

(ii) Prove that $h = \sqrt{ab}$, as given in line 24. **[2]**

This was generally proved correctly.

Question 15

15 It is given in lines 31–32 that the square has the smallest perimeter of all rectangles with the same area. Using this fact, prove by contradiction that among rectangles of a given perimeter, $4L$, the square with side L has the largest area. **[3]**

Proof by contradiction was challenging for the majority of candidates. Many offered no solution and some tried a proof by deduction. Those candidates that were successfully were able to make progress from a clear, suitable initial statement for contradiction.

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