OCR
Oxford Cambridge and RSA

GCE

Mathematics

Unit **4723**: Core Mathematics 3

Advanced GCE

Mark Scheme for June 2018

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1			Attempt calculation $k(y+4y+2y+...)$	M1	Any constant k; using y values with coefficients 1, 2, 4 each occurring at least once; allow M1 for attempt using y values based on wrong x values; for incorrect or missing brackets, give M0 unless subsequent calculation shows correct interpretation; allow M1 for 1, 2, 4, 2, 1 pattern of coefficients; allow M1 if more than 4 strips used; if decimal values used immediately, allow M1 if values given to only 1dp
			Obtain $\frac{1}{3}(e^2+4e+2e^{\frac{2}{3}}+4e^{\frac{2}{4}}+e^{\frac{2}{5}})$	A1	Or with decimal values given to at least 2 dp; e.g. $\frac{1}{3}(7.389+4\times2.718+2\times1.948+4\times1.649+1.492)$
			Obtain 10.1	A1	Allow 10.08 or more decimal places provided value rounds to 10.1; $10.1+c$ or equiv is A0; Answer only with no working seen is 0/3
				[3]	
2			Attempt process for finding 2 critical values	M1	<u>Either</u> Squaring both sides to obtain 3 terms on each side, simplifying and attempting to find critical values <u>Or</u> Attempting to solve 2 linear equations/inequalities, one with signs of 4x and x the same and the other with signs different (but condoning other sign errors)
			Obtain $-\frac{11}{3}$	A1	Award whether given as $x=-\frac{11}{3}$ or $x<-\frac{11}{3}$ or $x>-\frac{11}{3}$ or …
			Obtain 1	A1	Award whether given as $x=1$ or $x<1$ or $x>1$ or …
			State answer of form $a<x<b$ where a and b are their critical values (and $a<b$)	M1	Conclusion $x<a, x<b$ earns M0; allow $\le$ signs for the method mark; ignore incorrect sketch
			Obtain $-\frac{11}{3}<x<1$	A1	Or equiv; logically correct "$x>-\frac{11}{3}$ and $x<1$" is M1A1 but "$x>-\frac{11}{3}, x<1$" is M0A0; one or more $\le$ signs means A0
				[5]	

Question			Answer	Marks	Guidance
3	i		Draw (more or less) correct curve in first and fourth quadrants	B1	Showing increasing curve; no need to indicate asymptote; curve should not touch y-axis; ignore stated x-intercept; ignore equation of asymptote; allow flattening of curve but B0 if there is a clear maximum point
			Equate $2x-a$ to 1 or e^0 and attempt soln	M1	M0 if particular value for a used; [Or: for M1, state translation x-direction by a then stretch x-dir'n sf $\frac{1}{2}$ or state stretch x-dir'n sf $\frac{1}{2}$ then translation x-dir'n by $\frac{1}{2}a$]
			Obtain $\frac{1}{2}(a+1)$	A1 **[3]**	Or equiv but with e^0 replaced by 1
3	ii		Differentiate to obtain $\dfrac{6}{2x-a}$	B1	
			Attempt gradient at P using derivative of form $\dfrac{k}{2x-a}$ and their x-coordinate at P	M1	Or unsimplified equiv
			Obtain $y=6x-3a-3$	A1 **[3]**	
4			Attempt use of quotient rule or equiv	M1	Condone one error only but must be subtraction in numerator with the two terms the right way round; condone absence of brackets; no denominator is M0; for pr rule attempt, condone one error but expect terms involving $(x^4+30)^{-1}$ and $(x^4+30)^{-2}$
			Obtain $\dfrac{4x(x^4+30)-4x^3(2x^2+1)}{(x^4+30)^2}$ or equiv	A1	Now with brackets as necessary
			Equate numerator to zero and attempt solution of 3-term quadratic equation in x^2	M1	Perhaps using substitution of $u=x^2$; ignore now any error in (or complete absence of) denominator of derivative; if numerator of QR was wrong way round, allow all marks now; solution using factors (where expansion gives at least two of three original terms) or formula (where correct values are substituted into correct formula)
			Obtain at least factors $(x^2-5)(x^2+6)$	A1	If using formula, obtain at least $x^2=5$ or $u=5$ or unsimplified equiv
			Obtain $(\sqrt{5},\ \frac{1}{5})$ or exact equiv	A1	
			Obtain $(-\sqrt{5},\ \frac{1}{5})$ or exact equiv	A1	SC: if no or wrong y-coords given, award one A1 (of the final two A1s) for $x=\pm\sqrt{5}$
			Obtain $(0,\ \frac{1}{30})$ or exact equiv	B1 **[7]**	Award irrespective of rest of solution; award B0 if solution all correct but extra points given

Question			Answer	Marks	Guidance
5	i		State $y > -3$ or $f(x) > -3$	B1 **[1]**	Not $\geq$; accept $f > -3$ or precise equiv in words but not $x > -3$; allow $-3 < y < \infty$
5	ii		Obtain expression involving $\ln(y+3)$ or $\ln(x+3)$ Obtain correct $\frac{1}{2}\ln(x+3)$	M1 A1 **[2]**	Allow M1 for appearance of $\ln y + 3$ or $\ln x + 3$ Now with brackets (or modulus signs); accept $\ln\sqrt{x+3}$
5	iii		Stretch parallel to x-axis with factor $\frac{1}{2}$ Translation parallel to y-axis by 3 down Reflection in $y = x$	B1 B1 B1 **[3]**	Accept 'in x-direction' but not 'in x-axis' nor 'along x-axis'; B0 for 'squeeze factor 2' Accept 'in y-direction'; accept equiv in terms of vector; accept by 3 in negative y-direction; B0 for 'double negative' of -3 downwards; B0 for contradictory statement; accept alternative such as 'move' or 'shift' if referring to translation Accept 'along $y = x$' [Accept these transformations given in different order provided clear which is which]
5	iv		Sketch (more or less) correct curve with correct curvatures State answer of form $a < k < b$ where either $a = 0$ or $b = 3$ or both State $0 < k < 3$	B1 M1 A1 **[3]**	Must exist for $x < 0$; no need for intercepts or equations of asymptotes to appear; condone slight smoothing of cusp on x-axis and hint of maximum point but B0 if either very pronounced Allow $\leq$ sign(s) here; allow use of y instead of k here Must be k now; accept the logically correct '$k > 0$ and $k < 3$'
6	a		Differentiate V to obtain form $k(5+2x)^2$ Obtain correct $12(5+2x)^2$ or (unsimplified) equiv Divide 15 by their attempt at $\frac{dV}{dx}$ with $x = 1.6$ Obtain 0.019 or 0.0186	M1 A1 M1 A1 **[4]**	M0 if -250 retained in derivative If V expanded, $300 + 240x + 48x^2$ earns B2, and award B1 if this has only one error Not dependent on first M but attempt at differentiation must have occurred Accept greater accuracy; no need for units unless adjustment made in which case units must be clearly shown (e.g. 0.00031 m s^{-1})

Question			Answer	Marks	Guidance
6	b	i	State $Ae^{15\lambda}=48$ State $A\lambda e^{15\lambda}=1.2$ Obtain $\lambda=0.025$ Obtain $A=33$	B1 B1 B1 B1 **[4]**	 Or greater accuracy 32.989…; or exact equiv $48e^{-\frac{3}{8}}$
		ii	Attempt to find t using correct procedure Obtain 30.1	M1 A1 **[2]**	At least as far as appearance of $\ln(70/A)$ whether values for A, λ substituted or not Or value rounding to 30.1
7	i		Use $\tan 2x=\dfrac{2t}{1-t^2}$ Use correct process to simplify at least as far as polynomial equation in t or $\tan x$ Confirm $t=\sqrt[3]{\frac{1}{2}+\frac{1}{4}t-\frac{1}{2}t^2}$ with t used now	B1 M1 A1 **[3]**	Perhaps with $\tan x$ instead of t at this stage Assuming that one term has denominator of form $a+bt^2$; correct equation is $8t^3+4t^2-2t-4=0$ (or equiv) AG; with sufficient detail seen and no error noted; allow correct attempt where expressions meet 'in the middle'
7	ii		Evaluate appropriate expression from part (i) for both values 0.7 and 0.8 Obtain –0.05… (for 0.7) and 0.07… (for 0.8) and draw attention to sign change	M1 A1 **[2]**	Evidence of calculation must be there – merely saying one negative and one positive is M0; if no explicit working seen, M1 is implied by at least one correct value but, if both wrong, it is M0 Or equiv such as observe 0.7 < 0.75 but 0.8 > 0.72 if calculation involves only RHS
7	iii		Carry out process starting with 0.75 to produce at least four iterates Obtain at least four correct iterates, to at least 3 dp (rounded or truncated) Obtain final answer 0.7428	M1 A1 A1 **[3]**	With evidence of correct formula being used $0.75 \to 0.740624 \to 0.743435 \to 0.742600 \to 0.742849 \to 0.742775$; condone slip in one or more iterates provided at least four correct steps taken Final answer needed to precisely 4 dp; answer must be indicated in some way, not merely appear as the final value in a sequence of iterates

Question			Answer	Marks	Guidance
7	iv		Solve $\tan 2y =$ (their answer from part iii) using correct sequence of operations Obtain 0.319	M1 A1 **[2]**	Or using more accurate version of answer from part (iii) Or greater accuracy rounding to 0.319; ignore extra values outside 0 to $\frac{1}{2}\pi$ but A0 if extra answers inside 0 to $\frac{1}{2}\pi$; solution using degrees can earn M1A0
8	a		Use, at some stage, $\operatorname{cosec}^2\alpha = 1 \div \sin^2\alpha$ Use identities to expand left-hand side Obtain $3\sin\alpha$ Obtain $\sin\alpha = \sqrt[3]{\frac{1}{3}}$ or exact equiv and no errors seen earlier in solution	B1 M1 A1 A1 **[4]**	Or equiv Allow exact values, approximate values, or $\sin 30$, etc. retained; allow benefit of doubt in respect of sign in expansion of $\cos(x+30)$ Final answer with $\pm$ is A0; ISW to find α
8	b		State $\sin 4\beta = 2\sin 2\beta\cos 2\beta$ Use $\sec^3\beta = 1 \div \cos^3\beta$ Use correct identity for $\cos 2\beta$ Attempt to express equation in terms of $\sin\beta$ only with no major errors of process Obtain $\sin^2\beta - 2\sin\beta - 1 = 0$ or 3-term equiv Attempt solution of 3-term quadratic equation in $\sin\beta$ Obtain, finally, $\sin\beta = 1 - \sqrt{2}$ only	B1 B1 B1 *M1 A1 M1 A1 **[7]**	 Either $1 - 2\sin^2\beta$ or $2\cos^2\beta - 1$ or $\cos^2\beta - \sin^2\beta$ Dependent on three B marks; earned as soon as equation involves $\sin\beta$ only Dependent *M Or exact equiv

Question			Answer	Marks	Guidance
9	i		<u>Either</u> Integrate to obtain correct $2\ln(2x+1)$	B1	Or, with substitution, $2\ln u$
			Use limits 0 and $\frac{7}{2}$ to obtain $2\ln 8$	B1	Allow $\ln 1$ retained at this stage; use limits 1 and 8 if using substitution
			Subtract area of rectangle from attempt at integral and use appropriate log property to obtain expression involving $\ln 2$	M1	
			Obtain $6\ln 2 - \frac{7}{4}$	A1	Or exact equiv involving $\ln 2$; ISW if subsequent multiplication by 4
			<u>Or</u> Rearrange equation to obtain $x = \dfrac{2}{y} - \dfrac{1}{2}$	B1	
			Integrate to obtain $2\ln y - \frac{1}{2}y$	B1	
			Use limits $\frac{1}{2}$ and 4 correctly for integral of form $a_1 \ln y + a_2 y$ and use appropriate logarithm property	M1	
			Obtain $6\ln 2 - \frac{7}{4}$	A1 **[4]**	Or exact equiv involving $\ln 2$; ISW if subsequent multiplication by 4

Question			Answer	Marks	Guidance
9	ii		Attempt to express x in terms of y	M1	Obtaining expression of form $\frac{a_3}{y}+a_4$ or equiv
			Obtain $x^2=\frac{4}{y^2}-\frac{2}{y}+\frac{1}{4}$ or $\frac{16-8y+y^2}{4y^2}$	A1	Or equiv with $(\ldots)^2$ expanded; at this stage with or without π
			Attempt reasonable integration of at least two terms of form $\frac{k_1}{y^2}+\frac{k_2}{y}+k_3$	M1	For non-zero constants k_1, k_2, k_3
			Obtain correct $-\frac{4}{y}-2\ln y+\frac{1}{4}y$	A1	At this stage with or without π
			Use limits $\frac{1}{2}$ and 4 to evaluate integral involving at least two terms of form $\frac{k_4}{y}+k_5\ln y+k_6y$	M1	For non-zero constants k_4, k_5, k_6
			Obtain $-2\ln 4+8+2\ln\frac{1}{2}-\frac{1}{8}$	A1	Or unsimplified equiv; at this stage with or without π
			Obtain final answer $\frac{63}{8}\pi-6\pi\ln 2$	A1	Or $\pi(\frac{63}{8}-6\ln 2)$ or equiv of required form; final A0 if subsequent multiplication by 8 to 'clear denominator'; if fully correct solution is followed by attempt at subtracting some cylinder, award final A0 [Attempt at rotation about the x-axis earns 0/7]
				[7]	

OCR (Oxford Cambridge and RSA Examinations)
The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored