Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4753**: Methods for Advanced Mathematics

Advanced GCE

Mark Scheme for June 2018

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

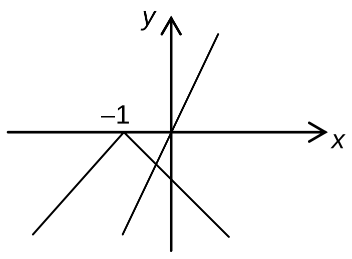
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		$3x^2 + 4y^2 = 4 \Rightarrow 6x + 8y\frac{dy}{dx} = 0$	M1	$8y\frac{dy}{dx}$ seen	or $6x\mathrm{d}x + 8y\mathrm{d}y = 0$
			$\Rightarrow \frac{dy}{dx} = -\frac{6x}{8y}$ $[= -\frac{3x}{4y}]$	A1	isw	
			OR			
			$y = \pm\frac{1}{2}(4-3x^2)^{\frac{1}{2}}$			
			$\Rightarrow \frac{dy}{dx} = (\mp)\frac{3}{2}x(4-3x^2)^{-\frac{1}{2}}$	B1	derivative correct (condone no ±)	
			$= -\frac{6x}{8y}$	B1 [2]	must deal with both signs for 2nd B1	
1	(ii)		When $x = 1$, $y = ½$	B1	o.e. e.g. √¼	or from $\frac{dy}{dx} = -\frac{3}{2}x(4-3x^2)^{-\frac{1}{2}}$ SCB2 but must be correct
			$\frac{dy}{dx} = -\frac{3}{2}$	B1	o.e (soi)	
			$\frac{dy}{dt} = \frac{dy}{dx}.\frac{dx}{dt}$	M1	o.e – any correct chain rule	
			$= -\frac{3}{2}\times 4 = -6$	A1cao [4]		
2			f: odd Y, even N, periodic N	B1	all correct	
			g: odd N, even N, periodic Y,	B1	all correct	
			period π	B1	allow 180°	or 0 to π or 180°
			h: odd N, even Y, periodic N	B1 [4]	all correct	
3	(i)		25(%)	B1 [1]	not ¼ or 0.25	
3	(ii)		$½ M_0 = M_0 \mathrm{e}^{-14.4k}$	M1	o.e. (e.g. taking a value for M_0, incl 1)	or $¼ M_0 = M_0 \mathrm{e}^{-28.8k}$ o.e.
			$\Rightarrow -14.4k = \ln(½)$	A1	o.e.	
			$\Rightarrow k = 0.048$ (2 s.f.)	A1cao [3]	mark final answer	

Question			Answer	Marks	Guidance	
4			$y = e^{-x}\sin 2x \Rightarrow \frac{dy}{dx} = e^{-x}.2\cos 2x - e^{-x}\sin 2x$	M1 A1	product rule used correct	of form $a e^{-x}\cos 2x + b e^{-x}\sin 2x$, where a and b are non-zero integers
			At P $2e^{-x}\cos 2x - e^{-x}\sin 2x = 0$	M1	their $\frac{dy}{dx} = 0$	
			$\Rightarrow 2\cos 2x = \sin 2x$, $\tan 2x = 2$ $\Rightarrow [2x = 1.107\ldots]$, $x = 0.554$	M1 A1	sin/cos = tan used art 0.55, not 31.7°	or squaring and using $\cos^2 2x = 1 - \sin^2 2x$
			$y = 0.514$	A1 [6]	art 0.51	
5	(i)			M1 A1 B1 [3]	$y = -\lvert x+1\rvert$ correct shape, apex on Ox apex at (–1, 0) indicated on sketch $y = 2x$ steeper gradient through origin, crossing $y = -\lvert x+1\rvert$ at one point	(inverted ‘v’)
5	(ii)		$-(x+1) = 2x$ $\Rightarrow x = -\frac{1}{3}$	M1 A1 [2]	or, from squaring, $4x^2 = x^2 + 2x + 1$ or –0.33 or better, if any additional solution (e.g. $x = 1$) given then A0.	or $x + 1 = -2x$ or $-x - 1 = 2x$ (must resolve the modulus)

Question			Answer	Marks	Guidance	
6	(i)		$g(\frac{1}{2}) = \frac{\pi}{6}$	B1	arcsin ½ = $\frac{\pi}{6}$ soi	condone 30° or 0.523…
			$h(\frac{1}{2}) = \frac{5\pi}{6}$	B1cao [2]	must be exact, allow $0.8\dot{3}\pi$	not 0.83π
6	(ii)		$h(x) = 2\arcsin x + \frac{\pi}{2}$	B1	or $2\sin^{-1} x + \frac{\pi}{2}$	
			$y = 2\arcsin x + \frac{\pi}{2}$ $x \leftrightarrow y$			may interchange x and y at any stage
			$x = 2\arcsin y + \frac{\pi}{2}$	M1	attempt to solve for y	… or x if not yet interchanged
			$\Rightarrow \frac{1}{2}x - \frac{\pi}{4} = \arcsin y$	A1	or $\frac{1}{2}y - \frac{\pi}{4} = \arcsin x$ o.e.	
			$\Rightarrow y = \sin(\frac{1}{2}x - \frac{1}{4}\pi)$ [so $h^{-1}(x) = \sin(\frac{1}{2}x - \frac{1}{4}\pi)$]	A1	oe e.g. $\sin\left(\frac{x-\pi/2}{2}\right)$ mark final answer	
			OR			
			$f^{-1}(x) = ½ (x - ½ \pi)$	B1		
			$g^{-1}(x) = \sin x$	B1		
			$h^{-1}(x) = g^{-1}f^{-1}(x)$	M1		
			$= \sin(\frac{1}{2}x - \frac{1}{4}\pi)$	A1 [4]		
7			$n^3 - 3n^2 + 2n = n(n^2 - 3n + 2)$	B1	or, e.g. $(n-1)(n^2 - 2n)$	
			$= n(n-1)(n-2)$	B1		
			n, $n-1$ and $n-2$ are consecutive integers	B1		
			one must be even, one must be a multiple of 3	B1	divisible by either 2 or 3	condone 'factor' for 'multiple'
			so it is divisible by 6	B1 [5]	both, and conclusion	

Question			Answer	Marks	Guidance	
8	(i)		$\dfrac{dy}{dx}=\dfrac{(1+x^{\frac{1}{2}})2.\frac{1}{2}x^{-\frac{1}{2}}-2x^{\frac{1}{2}}.\frac{1}{2}x^{-1/2}}{(1+x^{\frac{1}{2}})^2}$	M1 B1 A1	quotient or product rule (see right) allow 1 error (denom correct) derivative of $x^{1/2}=½x^{-1/2}$ soi correct expression – condone missing brackets	$(1+x^{\frac{1}{2}})^{-1}.2.\frac{1}{2}x^{-\frac{1}{2}}+2x^{\frac{1}{2}}.(-1)(1+x^{\frac{1}{2}})^{-2}\frac{1}{2}x^{-\frac{1}{2}}$ $=(1+x^{\frac{1}{2}})^{-1}x^{-\frac{1}{2}}-(1+x^{\frac{1}{2}})^{-2}=\dfrac{(1+x^{\frac{1}{2}})x^{-\frac{1}{2}}-1}{(1+x^{\frac{1}{2}})^2}$
			$=\dfrac{x^{-\frac{1}{2}}+1-1}{(1+x^{\frac{1}{2}})^2}=\dfrac{1}{\sqrt{x}(1+\sqrt{x})^2}$ *	A1	or $\dfrac{1}{x^{\frac{1}{2}}(1+x^{\frac{1}{2}})^2}$ **NB AG**	$=\dfrac{x^{-\frac{1}{2}}}{(1+x^{\frac{1}{2}})^2}=\dfrac{1}{\sqrt{x}(1+\sqrt{x})^2}$ *
			When $x=1$, $\dfrac{dy}{dx}=\dfrac{1}{4}$	B1		
			Equation of tangent is $y-1=\frac{1}{4}(x-1)$	B1ft	or $y=\frac{1}{4}x+c,\ 1=\frac{1}{4}+c,\ c=\frac{3}{4}$	ft their ¼
			$\Rightarrow x-4y+3=0$	B1cao [7]	or $4y-x-3=0$	
8	(ii)		$\dfrac{du}{dx}=\dfrac{1}{2}x^{-\frac{1}{2}}$ $x^{\frac{1}{2}}=u-1$	M1	or $du=\frac{1}{2}x^{-\frac{1}{2}}[dx]$	derivative must be correct
			$\displaystyle\int\frac{2\sqrt{x}}{1+\sqrt{x}}dx=\int\frac{2\sqrt{x}}{1+\sqrt{x}}\frac{1}{\frac{1}{2}x^{-\frac{1}{2}}}du$	M1	substituting for dx (must be correct)	
			$\displaystyle=\int\frac{4(u-1)^2}{u}du$ *	A1	with convincing working **NB AG**	
				[3]		

Question			Answer	Marks	Guidance	
8	(iii)		Area under curve $=\int_1^2 \frac{4(u-1)^2}{u}\,\mathrm{d}u$	B1	changing limits to 1 and 2	or using these at some stage
			$=\int_1^2 [4u-8+4/u]\,\mathrm{d}u$	M1	expanding and dividing by u	condone one error
			$=\left[2u^2-8u+4\ln u\right]_1^2$ =(8 – 16 + 4ln2) – (2 – 8) = 4ln2 – 2	A1 A1	$\left[2u^2-8u+4\ln u\right]$ o.e.	
			[Q is] (0, ¾) Area under line = $\frac{1}{2}(\frac{3}{4}+1)\times 1$ $=\frac{7}{8}$ or 0.875	B1ft M1 A1cao	soi, or $\int_0^1 \frac{1}{4}(x+3)\,\mathrm{d}x$ $=\frac{1}{4}\left[\frac{1}{2}x^2+3x\right]_0^1$ must correctly ft from their line equation in (i)	ft their line equation or rectangle ± triangle
			or $y=\frac{2\sqrt{x}}{1+\sqrt{x}} \Rightarrow x=\frac{y^2}{(2-y)^2}$ Area between curve and y-axis= $\int_0^1 \frac{y^2}{(2-y)^2}\,\mathrm{d}x$	M1 A1	Finding x in terms of y	
			letting u = 2 – y, $=\left[-4/u-4\ln u-u\right]_1^2$ = 3 – 4ln2	A1 A1cao	$=\left[-4/u-4\ln u-u\right]$ ignore limits	
			Q is (0, ¾) Area under triangle $=\frac{1}{2}\times\frac{1}{4}\times 1$ $=\frac{1}{8}$	B1 B1 A1		
			required area $=2\frac{7}{8}-4\ln 2$	B1cao [8]	o.e., must be exact	$=\frac{23}{8}-4\ln 2$ or $2.875-4\ln 2$

Question			Answer	Marks	Guidance	
9	(i)		Translation $\begin{pmatrix}2\\0\end{pmatrix}$ [One-way] stretch [parallel to] y-axis [scale factor] 2	B1 M1 A1 [3]	allow 'shift' '+2 in x-direction' or 'in y-direction'	not 'move' transformations can be in either order
9	(ii)		P is (3, 0) Q: $2\ln(x-2) = \ln(x-2)^2$ $\Rightarrow \ln x = \ln(x-2)^2$ $\Rightarrow x = (x-2)^2$ $\Rightarrow x^2 - 5x + 4 = 0$ $\Rightarrow x = 4$ $y = \ln 4$	B1 M1 A1 A1 A1 [5]	allow $x = 3$ or $\ln(x-2) = \ln x^{1/2}$… NB must be from correct work or 2ln 2, must be exact (but can isw)	
9	(iii)		let $u = \ln x$, $v' = 1$, $\Rightarrow du/dx = 1/x$, $v = x$ $\int \ln x\,dx = x\ln x - \int x.\frac{1}{x}dx$ $= x\ln x - \int dx = x\ln x - x + c$ *	M1 A1 A1 [3]	**NB AG** must see $x\ln x - \int [1]\,dx$	

Question			Answer	Marks	Guidance	
9	(iv)		Area under $y = \ln x$ $= [x\ln x - x]_1^4$ $= 4\ln 4 - 3$	B1ft B1	ft their 4 or 2.545…isw, if unsupported B0B0	
			Area under $y = 2\ln(x-2)$ is $\int_3^4 2\ln(x-2)\,dx$ let $u = x - 2$, $du = dx$ $= \int_1^2 2\ln u\,du$ $= [2u\ln u - 2u]_1^2$ $= 4\ln 2 - 2$	 M1 A1 A1	 substituting $u = x - 2$ $[2u\ln u - 2u]$ $4\ln 2 - 2$ or 0.772…isw unsupported M0	 or $[2(x-2)\ln(x-2) - 2(x-2)]$ B2
			or $\int 2\ln(x-2)\,dx = 2x\ln(x-2) - \int \frac{2x}{x-2}\,dx$ * let $u = x - 2$, $\int \frac{2x}{x-2}\,dx = \int \frac{2(u+2)}{u}\,du = \int 2 + \frac{4}{u}\,du$ $= [2u + 4\ln u]$ $\int_3^4 2\ln(x-2)\,dx = 8\ln 2 - [2u + 4\ln u]_1^2$ $= 4\ln 2 - 2$	 M1dep A1 A1	 substituting $u = x - 2$ dep * or 0.772…isw unsupported M0	 or $\int \frac{2x}{x-2}\,dx = \int 2 + \frac{4}{x-2}\,du$ $= [2x + 4\ln(x-2)]$
			Area is $\int_1^4 \ln x\,dx - \int_3^4 2\ln(x-2)\,dx$ $= 4\ln 4 - 3 - 4\ln 2 + 2 = 4\ln 2 - 1$ [so $m = 4$ and $n = -1$]	M1 A1cao [7]	ft their 3 and 4 from part (ii)	could be seen earlier

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