OCR
Oxford Cambridge and RSA

GCE

Mathematics

Unit **4733**: Probability and Statistics 2

Advanced GCE

Mark Scheme for June 2018

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

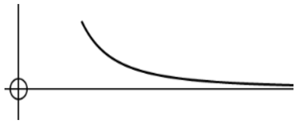
Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Question		Answer/Indicative content	Marks	Guidance
1		$\hat{\mu} = \bar{w} = \frac{555}{15}$ **= 37** $\frac{20808}{15} - \bar{w}^2$ [=18.2] $\times\frac{15}{14}$; **= 19.5**	B1 M1 M1 A1 **[4]**	37 only, must be stated separately, *not* isw If single formula used, give M1 for divisor 14 anywhere. 18.2 seen gets M1 Multiply by 15/14 Answer, 19.5 or exact equivalent, no working needed
2	**(i)**	Produces unbiased sample *or* allows theoretical calculations to be performed	B1 **[1]**	Or equivalent. Not *just* "sample is representative" or "quicker/cheaper" but do not penalise these if included as well. No wrong reasons.
	(ii)	Unbiased method described AND applied to given numbers to obtain at least 1 letter, e.g. 2 digits at a time, first or last 2 digits, ×26 and round Five letters obtained, no repeats	M1 A1 **[2]**	SC1: Random numbers not used consecutively or sequentially: M1A0 SC2: Biased method, e.g. digits combined, e.g. $1^{st} + 3^{rd}$: M0 SC3: Multiply by number other than 26 or 100 and then correct: M1A0 SC4: Systematic: M1A1 if random number used for starting point, else M0 SC5: Unbiased method but not clearly explained, 5 different letters: B1
3	**(i)**	1 – P(≤ 4) = **0.1371**	M1 A1 **[2]**	For 1 – P(≤ 4) or 1 – P(≤ 5) from Po(2.7, 2.6 or 2.8). *Not* 0.8629. Or 0.137. 1 – P(≤ 5) = 0.0567, also 0.1226, 0.1523, 0.0490, 0.0651: M1A0
	(ii)	B(4, 0.1371): ${}^4C_2 \times 0.1371^2 \times 0.8629^2$ = **0.084(0)**	M1 M1 A1 **[3]**	Use B(4, their answer to (i)) ${}^4C_2 \times p^2 \times (1 - p)^2$, any p, can be implied, independent of first M1 awrt 0.0840, *allow* from B(4, 0.8629), ***withhold if > 6 DP in final answer***
	(iii)	$e^{-10.8}\frac{10.8^{12}}{12!}$ = **0.107(24)**	M1 A1 **[2]**	Correct Poisson formula, their attempt at 4×2.7 or 4^3×2.7 Answer, a.r.t. 0.107 Answer only is 0

Question		Answer/Indicative content	Marks	Guidance
4	(i)	$E(Y) = \Sigma y P(Y = y)$ [= 1.1] $\text{Var}(Y) = \Sigma y^2 P(Y = y) - 1.1^2 = 2.3 - 1.1^2 = 1.09$ Normal, mean their 1.1 variance their $\sigma^2/50 = 0.0218$	M1 A1 M1 A1ft B1ft **[5]**	Allow if $\Sigma p(Y = y)$ wrongly evaluated. *Not* for 1.1/50 if this is used to find var Exact only, can be implied Expect to see N(1.1, 0.0218) FT on their E(Y), numerical value needed FT on their Var(Y), numerical value needed as final answer, but allow "1.09/50". Not from binomial unless explicitly "variance"
	(ii)	1.4, 1.42, 1.44, 1.46, 1.48, 1.5	B1 **[1]**	These only, but allow omission of 1.4 and 1.5
5	(i)	H_0: $\lambda = 6$, H_1: $\lambda \neq 6$ R ~ Po(6) where R is the number of mistakes α: $P(R \geq 10) = 1 - 0.9161 = 0.0839$ > 0.025	B2 M1 A1 A1	One error (e.g. >, wrong or no letter) B1, but r, x etc: B0 Po(6) stated or implied, e.g. N(6, 6) [but if Normal used, no more marks] $P(\geq 10) = 0.0839$, or $P(< 10) = 0.9161$ *Not* $P(\geq 10) + P(\leq 2)$ Compare $P(\geq 10)$ with 0.025 or $P(< 10)$ with 0.975
		β: CR is ≥ 12 [and ≤ 1] and $10 < 12$ $p = 0.0201$ [+ 0.0174 = 0.0375]	A1 A1	Correct CR stated, explicit comparison with 10 (if both tails used, must be √) This probability seen, a.r.t. 0.020. Award if 0.9799 seen and CR is correct. If CR not clearly stated or implied (e.g. by 10 < 12), cannot get last M1A1. *See exemplars.* SC 1-tailed: CR ≥ 11 and 10 < 11: A0A1
		Do not reject H_0. There is insufficient evidence that the average number of mistakes has changed.	M1 A1 **[7]**	Correct first conclusion, CR $\geq x$ from Po(6), *not* $P(> 10)$ [= 0.0426] or $P(\leq 10)$ [= 0.9574] or $P(= 10)$ [= 0.0413]. Allow from 0.9161 < 0.975 Interpreted, in context, acknowledge uncertainty, double negative. SC: Normal: max B2 M1 SC: Mix of methods: max B2 M1. Also for both unless both correct
	(ii)(a)	Mistakes must occur at constant average rate	B1 **[1]**	Must be contextualised (not "they occur", "events occur") Allow "uniform rate" but not "constant rate" nor "average constant rate". Not "equally probable at any time". No extras but ignore "singly"
	(ii)(b)	Teacher may become tired	B1 **[1]**	Any sensible reason for different average rate at different times, *not* in different sessions. *Not* e.g. "some reports are harder to write". Do not award if anything actually wrong seen. Ignore "singly".
	(ii)(c)	More information needed on whether/how the mean changes in the second hour/over a longer time interval	B1 **[1]**	Reason why answer to (ii)(b) means that more information is needed. E.g. "mean not proportional to the length of time". *Not just* statement of assumptions. Not just an answer to (ii)(a) or (ii)(b).

Question		Answer/Indicative content	Marks	Guidance
6	(i)	$T_0 = L$	B1 **[1]**	$T_0 = L$, or $T_0 \geq L$, stated or clearly implied. Not *just* "close to L", but "just above" is B1. No wrong extras such as "less than t" or "> 0". *Not* "$t = L$"
	(ii)	$\int_L^\infty kt^{-4}\mathrm{d}t = \left[-\frac{k}{3t^3}\right]_L^\infty = \frac{k}{3L^3}$ $= 1$ so $k = 3L^3$	M1 B1 A1 **[3]**	Attempt $\int \mathrm{f}(t)\, \mathrm{d}t$ and equate to 1, limits L and ∞ seen somewhere (if upper limit not given as ∞, must use different letter [*not* t] and state "take limit") Correct indefinite integral, allow $-\tfrac{1}{3}kt^{-3}$ Correctly obtain given answer. $\int_0^L kt^{-4}dt \to 3L^3$ is max B1 only
	(iii)	$\int_L^\infty t \times 3L^3t^{-4}\mathrm{d}t = \left[-\frac{3L^3}{2t^2}\right]_L^\infty = \frac{3L}{2}$ $\int_L^\infty t^2 \times 3L^3t^{-4}\mathrm{d}t = \left[-\frac{3L^3}{t}\right]_L^\infty = 3L^2$ Hence Var(T) = $3L^2 - \left(\tfrac{3}{2}L\right)^2 = \tfrac{3}{4}L^2$	M1 A1 M1 B1 M1 A1 **[6]**	Attempt $\int t\mathrm{f}(t)\, \mathrm{d}t$, limits dealt with correctly somewhere $\frac{3L}{2}$ or $\tfrac{1}{2}kL^{-2}$ seen or implied, www Attempt $\int t^2\mathrm{f}(t)\, \mathrm{d}t$, limits dealt with correctly *or* same limits as in mean Correct indefinite integral, allow $-k/t$ Subtract $[\mathrm{E}(T)]^2$ Www, *not* from [0, L], allow $0.75L^2$ [k not substituted: can get 5/6]
	(iv)	 No as graph not symmetrical	B1 B1 **[2]**	Starting to right of y-axis, clear attempt to be asymptotic to right but must be truncation not asymptote to left, labels not needed No with valid reason [*not* referring to CLT], e.g. "skewed". Ignore positive/ negative (skew). Needs roughly correct graph, no wrong reason seen. Allow "No as it is not bell-shaped". Any implied properties of normal (e.g. mean vs mode) must be justified

Question		Answer/Indicative content	Marks	Guidance
7	(i)	$\frac{58-\mu}{\sigma}=1$; $\frac{40-\mu}{\sigma}=-0.5$ or exact equivalent σ = **12** μ = **46**	M1dep* A1 B1 *M1 A1 A1 **[6]**	Standardise once and equate to Φ^{-1}, allow wrong sign, σ^2, 1–, cc etc, no "*n*" Both equations fully correct apart possibly from value of Φ^{-1} Both correct *z* values correct to 3 sf, allow +/– errors, can be implied Solve to find μ or σ, correct choice of add/subtract, dependent on first M1 σ correct, allow within ±0.05, *not* from σ^2 μ correct, allow within ±0.05, *allow* from σ^2 E.g.: 40 – μ = +0.5σ → μ = 22, σ = 36: M1A0B1M1A0A0, total 3/6
	(ii) (a)	H_0: μ' = 56 H_1: μ' < 56 where μ' is the (population) mean MER of the new brand	 B2ft B1 **[3]**	Or H_0: $\mu \geq 56$; ft on their numerical 10+μ. Their 46, or words used: B0B0(B1) One error, e.g. H_1: $\mu \neq 56$ or H_1: $\mu > 56$: B1. Any symbol is OK apart from *p* (max B1B0B1) and *x*, *x*bar, *t*, *t*bar: B0B0(B1) Independent. Allow their symbol other than *x*, $\bar{x}$ etc, but must have "mean" or "expected value" and MER or equivalent, allow "hubs". *Not* old brand. *Not* sample mean. Expect to see μ
	(b) α:	$z=\frac{(\mu+8.8)-(\mu+10)}{\sqrt{12^2/200}}$ = –1.414 [*p* = 0.0787] < –1.282 [*p* < 0.10]	M1 A1 A1	Standardise with √200, allow √ errors, allow cc, allow 10 – 8.8 *z* in range [–1.41, –1.42], or *p* in range [0.078, 0.079], allow 0.9213 only if compared with 0.9 (or 0.95 etc). Correct value implies M1 Compare with –1.282, or *p* with 0.1 [if *p* < 0.5] or 0.9 [if *p* > 0.5]
	β: (CV)	$10-1.282\sqrt{\frac{12^2}{200}}$ or $56-1.282\sqrt{\frac{12^2}{200}}$ = 8.91 or 54.91 8.8 < 8.91 or 54.8 < 54.91	M1 A1 A1ft	(μ +) 10 – *z*σ/√50, any recognisable *z*, allow √ errors etc, ignore 10 +, *not* 8.8 *z* = 1.282 and correct √ etc Compare (μ +) 8.91 (or better) with (μ +) 8.8, ignore (μ +) 10 + … SC: 2-tailed, 8.6 (54.6) gets M1A0A1ft M1A1
		Reject H_0. Significant evidence that mean MER of new brand is not (at least) 10 m more than that of long-established brand [e.g. "less than 56 m" or "manufacturer's claim is invalid"]	M1 A1ft **[5]**	Consistent, needs √200, like-with-like comparison, hypotheses *not* 8.8/54.8 Contextualised, acknowledge uncertainty, their *z*, conclusion must be correct way round even if H_1 is wrong – independent of hypotheses SC1: 2-tailed: can get (B1B0B1) M1A1B0 M1A1 max 2/3 + 4/5 SC2: $\bar{x}$ and μ confused consistently: max (B0B0B1) M1A1 A1 M0 SC3: N(22, 36): *z* = –0.4714, *p* = 0.3187, CV 6.736: (B3) M1A0A1 M1A1 *Can't* get final M1A1 if: 54.8 in H_0; 200 omitted; not like-with-like, including e.g. (54.8 – 46)/(12/√200) *Can* get final M1A1 if: wrong σ, two-tailed, √ or cc errors
	(iii)	No as (told to assume that) the parent distribution is normal	B1 **[1]**	"No" stated *and* reason given. No wrong extras! "No as the sample is large and the parent distribution is normal": B0 "No as the parent distribution is normal": B1 "No as the distribution is normal" B1 (BOD)

<table>
<tr><th colspan="2">Question</th><th>Answer/Indicative content</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>8</td><td>(i)</td><td>B(60, 0.04) ≈ Po(2.4)
$np < 5$ or "p small"
$n > 50$ or "n large"

P(≤ 4) = 0.9041 so P(≥ 5) = 0.0959
P(≤ 5) = 0.9643 so P(≥ 6) = 0.0357
Hence CR is $R \geq 6$</td><td>M1
B1
B1

M1
A1
A1*
*A1dep
[7]</td><td>State or imply Poisson, their 60 × 0.04
One criterion. Numerical values not needed in (i)
Second, consistent, criterion. "n > (number not 50)" is B0.
Allow 2 of one type and 1 of the other type
Find at least one cumulative Poisson probability > 0.5, can be implied
At least one of 0.9041, 0.9643, 0.0959, 0.0357
State "critical region is ≥ 6" [not just "critical value is 6"], any or no letter
Both relevant probabilities clearly seen, dep on clear statement of correct CR
SC Normal N(2.4, 2.304): 0
SC Exact binomial 0.0917 (0.9083) and 0.0325 (0.9675): 0</td></tr>
<tr><td></td><td>(ii)</td><td>100 × 0.0357 [= 3.57]</td><td>M1
[1]</td><td>100 × their upper-tail (and < 0.5) probability, allow "= 3.57 therefore 4", can be implied, e.g. by "Sig level 3.57% therefore expect 4".
Not 100 × 0.04, or 4 without working</td></tr>
<tr><td></td><td>(iii)</td><td>B(60, 0.15) ≈ N(9, 7.65)

n large or $np > 5$
p close to ½ or $nq = 51 > 5$

$P(\leq 5) = \Phi\left(\dfrac{5.5-9}{\sqrt{7.65}}\right) = \Phi(-1.265)$
= 0.1029</td><td>M1

A1
B1
B1

M1ft
A1ft
A1
[7]</td><td>Normal, their 60×0.15 Don't FT on their calculation if ≤ 5
SC: 100×0.15: (np = 15, npq = 12.75) can get M1A0B1B1M1A1A0
Variance 7.65 stated or implied, allow SD = 7.65
One condition
Second condition consistent with the first. No others or wrong. Not npq
Each numerical condition needs <u>value</u> of np, nq visible somewhere.
[Allow B2 if n or p wrong provided "np" is > 5]
Ignore "n > 30" but max B1B0 if any other criteria (e.g. npq) given
Standardise their CV if 5, 6 or 7, <u>final</u> answer < 0.5, np, npq, allow √ errors
Their 5.5 and √ both correct
<u>Final</u> answer, awrt 0.103, cwo, do not isw if 0.103 seen but final answer > 0.5
Wrong/no CC: 0.183, 0.139. CV 5: 0.0741, 0.0519, CV 7: 0.183, 0.235, 0.294
SC: Po(9) [0.1157] or exact binomial [0.0968]: M0B0 B3 max 3/7</td></tr>
<tr><td></td><td></td><td colspan="3">SC Misread p = 0.4 in (i):
8(i) N(24, 14.4), $24+1.645\sqrt{14.4} = 30.24$, CR $R \geq 31$ M1B1B1 M1A1A0 (MR) A1 (6/7)
8(ii) 100×0.05 M1 (1/1)
8(iii) N(9. 7.65), $z = 7.77$, $p = 1$ M0A0B0B0 M1A1A1 (3/7)</td></tr>
</table>

OCR (Oxford Cambridge and RSA Examinations)
The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; The Triangle Building, Shaftesbury Road, Cambridge, CB2 8EA
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553