

OCR

Oxford Cambridge and RSA

Monday 25 June 2018 – Morning

A2 GCE MATHEMATICS

4726/01 Further Pure Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4726/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

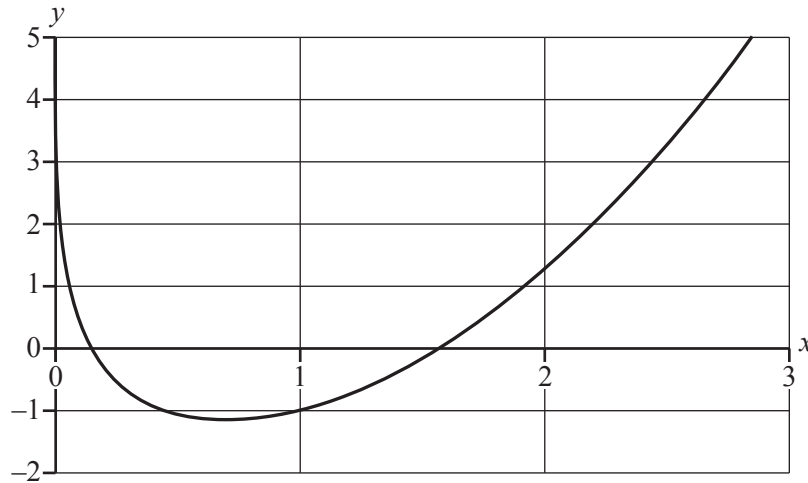
INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Answer **all** the questions.

- 1 The equation of a curve is $y = \frac{x^2 + 1}{(x-1)^2}$.
- (i) Write down the equations of the asymptotes. [2]
- (ii) Find the coordinates of the stationary point on the curve and hence determine the range of y . [6]
- (iii) Sketch the curve. Indicate the asymptotes in your sketch. [2]
- 2 (i) Using the definition of $\cosh x$ in terms of e^x and e^{-x} , show that $\cosh 2x = 2 \cosh^2 x - 1$. [2]
- (ii) Find $\int_0^1 \cosh^2 3x \, dx$, giving your answer in the form $A + B \sinh C$, where A , B and C are constants to be determined. [3]
- 3 The equation of a curve is $y = \cosh x - 2 \sinh 2x$.
- Show that the curve has no turning points. [5]
- 4 It is given that $I_n = \int_0^1 x^n e^{x-1} \, dx$ for $n \geq 1$.
- (i) Show that $I_n = 1 - nI_{n-1}$ for $n \geq 2$. [3]
- (ii) Find the exact value of I_4 . [4]
- (iii) For the curve $y = x^n e^{x-1}$, where $n \geq 1$, find $\frac{dy}{dx}$ and sketch the curve for $0 \leq x \leq 1$.
- Deduce that $0 < I_n < 1$ for all $n \geq 1$. [5]
- 5 By using the substitution $t = \tan \frac{1}{2}x$, find the exact value of $\int_0^{\frac{\pi}{2}} \frac{1}{\sin x + 1} \, dx$. [5]
- 6 It is given that $y = \tan^{-1} 3x$.
- (i) Using the derivative of $\tan^{-1} x$ given in the List of Formulae (MF1), find $\frac{dy}{dx}$. [2]
- (ii) Hence show that $(1 + 9x^2) \frac{d^3 y}{dx^3} + 36x \frac{d^2 y}{dx^2} + 18 \frac{dy}{dx} = 0$. [4]
- (iii) Find the Maclaurin series for y up to and including the term in x^3 . [3]

- 7 You are given that the equation $x^2 - \ln x - 2 = 0$ has two roots, α and β , where $0 < \alpha < 1$. The diagram shows a sketch of the curve $y = x^2 - \ln x - 2$.



- (i) The Newton-Raphson method is to be used to find α .
Explain what happens when the initial values $x_1 = 0.5$ and $x_1 = 1$ are used. [2]

- (ii) Show that the Newton-Raphson iterative formula for this equation can be written in the form

$$x_{r+1} = \frac{x_r(x_r^2 + \ln x_r + 1)}{(2x_r^2 - 1)} \quad [3]$$

- (iii) Using the initial value $x_1 = 0.1$, find x_2 and x_3 . Find α correct to 5 decimal places. [2]

- 8 You are given that the equation $x^3 + x - 4 = 0$ has a root, α , where $1 < \alpha < 2$.

The iterative formula $x_{r+1} = \sqrt[3]{(4-x_r)}$ with $x_1 = 1.4$ is to be used to find α .

- (i) Demonstrate that the iterative values converge to α by sketching a cobweb diagram. [2]

- (ii) Use the iterative formula to find α correct to 4 decimal places. Show the result of each step of the iteration correct to 5 decimal places. [3]

- (iii) The error, e_n is defined by $e_n = \alpha - x_n$.

Using the value of α found in part (ii), evaluate $\frac{e_3}{e_2}$ and $\frac{e_4}{e_3}$.

Comment on the values of $\frac{e_3}{e_2}$ and $\frac{e_4}{e_3}$ in relation to the gradient of the curve $y = \sqrt[3]{(4-x)}$ at $x = \alpha$. [3]

9 A curve has polar equation $r = 2 \cos \theta - 1$ for $-\frac{\pi}{3} \leq \theta \leq \frac{\pi}{3}$.

(i) Show that the line $\theta = 0$ is a line of symmetry. [2]

(ii) Sketch the curve, indicating

- the point on the curve where $\theta = 0$,
- the tangents at the pole.

[4]

(iii) Find the area enclosed by the curve.

[5]

END OF QUESTION PAPER

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