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A2 GCE MATHEMATICS (MEI)

4754/01B Applications of Advanced Mathematics (C4) Paper B: Comprehension

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Rain Stopped Play

Introduction

This article concerns the use of mathematical procedures to try to ensure a fair outcome in cricket matches where bad weather has reduced the time available for the match to be completed.

Cricket is a bat-and-ball game involving two teams, referred to here as Team 1 and Team 2, who take it in turn to try to score the most 'runs'. Runs in cricket correspond to points or goals in other games.

In the first part of a match, Team 1 is the 'batting' team trying to score runs while Team 2 is the 'fielding' team trying to prevent runs being scored. In the second part, the teams exchange roles so that Team 2 becomes the batting team and Team 1 becomes the fielding team. Each part of the game is called an 'innings', so that the first part of a game is Team 1's innings and the second part is Team 2's innings.

During an innings, a player from the fielding team (the 'bowler') bowls the ball to a player from the batting team (the 'batsman'). Each time the ball is bowled, the batsman has the opportunity of hitting the ball in such a way as to be able to score runs, while the bowler has a chance of dismissing the batsman. A batsman who is dismissed can take no further part in the team's innings, and the batting team is said to have 'lost a wicket'. At each loss of a wicket the dismissed batsman is replaced by another member of the batting team, until the team's 10th wicket is lost. At this point no more replacement batsmen are allowed, and the innings of the batting team is complete.

Limited Overs Cricket

In the form of cricket discussed in this article, the length of a team's innings is also limited by the number of times that the fielding team is allowed to bowl the ball. This is expressed as a number of 'overs', where an over refers to a particular bowler bowling the ball six times in succession. (This is usually referred to as the bowler 'bowling six balls', although the same actual cricket ball is in fact used each time.) So in a '50-over' match, for example, each team's innings can have at most $50 \times 6 = 300$ balls bowled. An innings will be shorter if a team loses all its 10 wickets before the maximum number of allowed overs have been bowled.

Matches in limited overs cricket have at most 50 overs per innings, but some matches are shorter, for example 45 or 40 overs per innings.

When bad weather interrupts the innings of either team it may be necessary to reduce the total number of overs that are available in the match, and so a revised target for the number of runs that Team 2 needs to score in order to win may be required. The number of runs that a team has scored and the number of wickets that it has lost when interruptions occur are both factors that need to be taken into account when the match resumes. These two factors are presented in a standard notation so that 127/4, for example, indicates that a team has scored 127 runs for the loss of 4 wickets.

Various different methods have been used to tackle this issue of setting a revised target when bad weather interrupts a match. One of these methods is the Average Run-Rate method.

Average Run-Rate (ARR) method

The Average Run-Rate (ARR) method was used at the start of limited overs cricket in the 1960s to set a target number of runs for Team 2 if bad weather reduced the time available for a game.

For example, if Team 1 scores 247 runs in 50 overs, then this gives an ARR of

$$\frac{247}{50} = 4.94 \text{ runs per over.}$$

If Team 2's innings is reduced to 33 overs, the target score for Team 2 would be $4.94 \times 33 = 163.02$.

As this is a non-integer value Team 2 would require 164 runs to win. If the target score had been an integer n then Team 2 would have required $n + 1$ runs to win.

It was generally felt that this method was not completely fair, as achieving a given average run rate for a smaller number of overs, without regard to the number of wickets that could be lost, was an easier task for Team 2. However, the method's simplicity, and a lack of any viable alternative, meant that the ARR method was used until the early 1990s.

One method that was tried as an alternative to the ARR method was the Most Productive Overs (MPO) method. In this method, Team 2 has a harder task as their target is equivalent to the total scored by Team 1 in its n highest scoring overs, where n is the number of overs available to Team 2.

The deficiency of this method was highlighted in a match between England and South Africa in 1992. In the closing stages of the match South Africa required 22 runs to win with 13 balls remaining. Rain stopped play for 12 minutes which meant that only 1 further ball could be bowled. The revised target under the MPO method, was 21 runs from 1 ball, which was an impossible target given that the maximum score from one ball is generally 6 runs.

After this match the International Cricket Council (ICC) appealed to mathematicians to come up with something better. Two British mathematicians, Frank Duckworth and Tony Lewis, took up this challenge and their method, known as the Duckworth-Lewis (D/L) method, was first used in 1997.

Duckworth-Lewis (D/L) method

The D/L method is based on the idea that, in building up its score of runs, a team has two 'resources' available: the number of wickets remaining and the number of overs to be bowled. A team's available resources decrease as its innings progresses as there are progressively fewer overs left to be bowled and wickets will usually be lost also.

Let R1 be the total resources used by Team 1 in their innings and let R2 be the total resources available to Team 2 in their innings. R1 and R2 take values between 0 and 100 and are the percentages of the total resources available to a team at the start of a 50-over match.

If S1 is the number of runs scored by Team 1, then the D/L method sets the target number of runs required by Team 2 to be S2, where

$$\text{S2} = \text{S1} \times \frac{\text{R2}}{\text{R1}} \text{ when R2} < \text{R1}, \qquad (1)$$

$$\text{S2} = \text{S1 when R2} = \text{R1}, \qquad (2)$$

$$\text{S2} = \text{S1} + \left(\frac{\text{R2} - \text{R1}}{100}\right) \times \text{G50 when R2} > \text{R1, where G50 is a constant.} \qquad (3)$$

Equation (1) deals with the case when Team 2's resources are less than Team 1's and the target for Team 2 is reduced in proportion to the resources.

Equation (2) indicates that no change in target is required when resources are equal.

Equation (3) applies when Team 2 has more resources available than Team 1, and their target score is increased by an amount proportional to the extra resources. The constant G50 in this equation is the average number of runs scored in a full 50-over innings at the appropriate level of the game, so that the increase in the target for Team 2 is the number of runs that an average team could expect to score with that amount of resources. As of 2014, the value of G50 in international matches is 245, and unless otherwise stated this value will be used in calculations in this article.

As before, if the target score for Team 2 is a non-integer, then Team 2's target to win is rounded up to the next integer. If the target score for Team 2 is an integer n, then Team 2 needs to score $n + 1$ runs to win.

For example, if Team 2 has only 70% of the resources that were available to Team 1, and Team 1 scored 273 then

$$\text{S2} = 273 \times 0.7 = 191.1,$$

so Team 2's target to win is 192.

A single table gives the resources remaining at any stage of an innings for any combination of overs remaining and wickets lost. The resources are expressed as percentages of the resources available at the start of a full 50-over innings. An abbreviated version of the D/L resource table is shown in Table 1.

Overers left	Wickets lost									
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
50	100	93.4	85.1	74.9	62.7	49.0	34.9	22.0	11.9	4.7
49	99.1	92.6	84.5	74.4	62.5	48.9	34.9	22.0	11.9	4.7
48	98.1	91.7	83.8	74.0	62.2	48.8	34.9	22.0	11.9	4.7
47	97.1	90.9	83.2	73.5	61.9	48.6	34.9	22.0	11.9	4.7
46	96.1	90.0	82.5	73.0	61.6	48.5	34.8	22.0	11.9	4.7
45	95.0	89.1	81.8	72.5	61.3	48.4	34.8	22.0	11.9	4.7
44	93.9	88.2	81.0	72.0	61.0	48.3	34.8	22.0	11.9	4.7
43	92.8	87.3	80.3	71.4	60.7	48.1	34.7	22.0	11.9	4.7
42	91.7	86.3	79.5	70.9	60.3	47.9	34.7	22.0	11.9	4.7
41	90.5	85.3	78.7	70.3	59.9	47.8	34.6	22.0	11.9	4.7
40	89.3	84.2	77.8	69.6	59.5	47.6	34.6	22.0	11.9	4.7
35	82.7	78.5	73.0	66.0	57.2	46.4	34.2	21.9	11.9	4.7
30	75.1	71.8	67.3	61.6	54.1	44.7	33.6	21.8	11.9	4.7
25	66.5	63.9	60.5	56.0	50.0	42.2	32.6	21.6	11.9	4.7
24	64.6	62.2	59.0	54.7	49.0	41.6	32.3	21.6	11.9	4.7
23	62.7	60.4	57.4	53.4	48.0	40.9	32.0	21.5	11.9	4.7
22	60.7	58.6	55.8	52.0	47.0	40.2	31.6	21.4	11.9	4.7
21	58.7	56.7	54.1	50.6	45.8	39.4	31.2	21.3	11.9	4.7
20	56.6	54.8	52.4	49.1	44.6	38.6	30.8	21.2	11.9	4.7
19	54.4	52.8	50.5	47.5	43.4	37.7	30.3	21.1	11.9	4.7
18	52.2	50.7	48.6	45.9	42.0	36.8	29.8	20.9	11.9	4.7
17	49.9	48.5	46.7	44.1	40.6	35.8	29.2	20.7	11.9	4.7
16	47.6	46.3	44.7	42.3	39.1	34.7	28.5	20.5	11.8	4.7
15	45.2	44.1	42.6	40.5	37.6	33.5	27.8	20.2	11.8	4.7
10	32.1	31.6	30.8	29.8	28.3	26.1	22.8	17.9	11.4	4.7
5	17.2	17.0	16.8	16.5	16.1	15.4	14.3	12.5	9.4	4.6

Table 1

As can be seen from the table, the resources available when overs are lost for a team batting second are not simply proportional to the number of overs lost. For example, in a 50-over match where Team 2 loses 10 overs at the start of their innings, the table gives 89.3 as the (percentage) resources for a team with 40 overs remaining and no wickets yet lost. This is significantly greater than the 80% that would correspond to the ARR method. So the D/L method can set a higher target for Team 2 than the ARR method, thus attempting to remove the unfairness shown by ARR to Team 1.

Table 1 is used for all matches, not only full 50-over ones. In a 40-over match, for example, Team 1 starts its innings with a resource value of 89.3, just as though 10 overs had been lost at the start of a full-length match.

The values in Table 1 were calculated from a mathematical formula giving the average number of runs, z, obtainable when a team has w wickets remaining with u overs left. The original formula developed was

$$z = z_0\left(1 - e^{-bu}\right), \quad (4)$$

though there have been some subsequent modifications. In equation (4), both b and z_0 are functions of w, though the actual definitions of these functions have never been revealed, due to commercial confidentiality.

Details of the how the D/L method works are shown in the following examples, which illustrate some of the ways in which interruptions can occur.

Team 1's innings complete; Team 2's innings cut short

45 overs per innings

Team 1 score 298 in their innings

Rain reduces Team 2's innings to 35 overs

In this scenario Team 1 started their innings with 45 overs and no wickets lost so according to Table 1, Team 1 began with 95.0% of the resources they would have had in a 50-over innings. Team 1 lost no resources due to the interruption so R1 = 95.0. The value of S1 is 298.

Team 2 start their innings with 35 overs and no wickets lost and so according to Table 1, R2 = 82.7.

R2 < R1, so using equation (1)

$$\text{S2} = 298 \times \frac{82.7}{95.0} = 259.416\ldots$$

and Team 2 are therefore set a target of 260 runs to win.

Team 1's innings cut short; Team 2's innings complete

50 overs per innings

Team 1 score 165/4 in 35 overs

It rains, leaving only enough time for a further 35 overs in the match

Team 2 must be set a target for their 35 overs

In this scenario Team 1 started their innings with 100% of their resources. When the rain came Team 1 had 15 overs remaining and had lost four wickets. According to Table 1 they had 37.6% resources available at that point, so they had only used up 62.4% of the original amount in scoring their 165 runs. The values for Team 1 are therefore R1 = 62.4 and S1 = 165.

Team 2 started their innings with only 35 overs and no wickets lost and so according to Table 1 they had 82.7% of the resources they would have had if they had their full complement of 50 overs so R2 = 82.7. In this case R2 > R1 and so equation (3) is used to set the target for Team 2. Using G50 = 245 gives

$$\text{S2} = 165 + 245\left(\frac{82.7 - 62.4}{100}\right) = 214.735.$$

So Team 2 are set a target of 215 runs to win.

Team 1's innings interrupted; Team 2's innings completed

45 overs per innings

Team 1 scores 172/5 in 30 overs and then rain causes the match to be reduced to 40 overs per team

Team 1 scores 220/7 in 40 overs

Team 2 must be set a target for their 40 overs

In this scenario Team 1 started their innings with 95.0% of their resources. When the rain came Team 1 had 15 overs left and had lost 5 wickets. Therefore Team 1 had 33.5% resources available at that point. When play resumed Team 1 had only 10 overs left, still with 5 wickets lost, and so they now only had 26.1% resources available. So the rain delay resulted in a loss of resources of $33.5 - 26.1 = 7.4\%$. Therefore the value of R1, the resources used by Team 1, is $95.0 - 7.4 = 87.6$.

Team 2 started their innings with 40 overs and no wickets lost and so $\text{R2} = 89.3$. In this case $\text{R2} > \text{R1}$ and so the score, S2, required by Team 2 as given by equation (3) is

$$\text{S2} = 220 + 245\left(\frac{89.3 - 87.6}{100}\right) = 224.165.$$

So Team 2 are set a target of 225 runs to win.

Along with the cases illustrated above, the D/L method can be used in a range of other scenarios. For example, matches being prematurely abandoned and multiple interruptions in either team's innings can be dealt with. There are also cases where the D/L method sets a target that requires Team 2 to score *fewer* runs than Team 1 in the same number of overs. This seems surprising at first sight, but is in fact an indication that Team 1 performed rather poorly in their innings.

In Conclusion

The version of the D/L method seen in this article is relatively transparent and easy to implement and was used widely in all forms of the game until 2003 but it unfortunately had a flaw when handling very high first innings scores. Over the last decade or so the D/L method has evolved so that it requires a computer to perform the calculations required to deal with this flaw but the version seen in this article is still used in most school and club games where computers are not widely available.