GCE

Mathematics (MEI)

Unit **4762**: Mechanics 2

Advanced GCE

Mark Scheme for June 2017

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand

a   Annotations should be used whenever appropriate during your marking.

      The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c   The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d   When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e   The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

         Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f   Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

         We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

         When a value is given in the paper
         Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

         When a value is not given in the paper
         Accept any answer that agrees with the correct value to 2 s.f.

         ft should be used so that only one mark is lost for each distinct error made in the accuracy to which working is done or an answer given. Refer cases to your Team Leader where the same type of error (e.g. errors due to premature approximation leading to error) has been made in different questions or parts of questions.

         There are some mistakes that might be repeated throughout a paper. If a candidate makes such a mistake, (eg uses a

calculator in wrong angle mode) then you will need to check the candidate's script for repetitions of the mistake and consult your Team Leader about what penalty should be given.

There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g   Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h   For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i   If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j   If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	Guidance	
1		**(i)**	The total LM of 0 is conserved	M1	Both 'LM = 0' and 'conserved' stated or implied in the answer, e.g. $6 \times 2 - 4 \times 3 = 0$ and 'conserved'	
			Cannot both be in same direction (nor in original direction)	A1 **[2]**	Accept "They must be going in opposite directions"	
		(ii)	$\rightarrow$+ve and 'after' velocities $\leftarrow v_A$ and $\rightarrow v_B$			
			PCLM $6 \times 2 + 4 \times -3 = -6v_A + 4v_B$	M1	Use of PCLM. Accept LHS as 0 without comment	
			so $\quad 3v_A = 2v_B$	A1	oe	
			NEL $\quad \dfrac{v_B - (-v_A)}{-3 - 2} = -e$	M1	Use of NEL. Must be attempt at separation/approach (right way up)	
			so $\quad v_B + v_A = 5e$	A1	oe (consistent with PCLM signs)	
			Solving, $v_A = 2e$ and $v_B = 3e$	A1 **[5]**	cao both, speeds not velocities required (v_A is AG)	
		(iii)	**either** Using A: $\leftarrow 6(2e + 2) = 12(e + 1)$	B1	cao oe	
			or Using B: $\rightarrow 4(3e + 3) = 12(e + 1)$	B1 **[1]**	cao oe	
		(iv)	**either** ▪Speed of B unchanged by impacting barrier; ▪B returns to its original position after 6 s; ▪in this time A has moved $12e$; ▪relative speed after 6 s is e.	B1ft B1ft	2 of these 4 statements made or implied, ft speed of B from (ii) all of these 4 statements made or implied, ft speed of B from (ii)	
			Time taken is 6 s + time to catch up (12 s) so 18 s	B1	cao	
			or ▪Speed of B unchanged by impacting barrier; ▪B travels distance $9e$ to barrier; ▪A is now $15e$ from barrier ▪relative speed after 6 s is e.	B1ft B1ft	2 of these 4 statements made or implied, ft speed of B from (ii) all of these 4 statements made or implied, ft speed of B from (ii)	
			Time taken is 3 s + time to catch up (15 s) so 18 s	B1	cao	

Question			Answer	Marks	Guidance
				[3]	
		(v)	Vel of B before impact is v at α to barrier; vel of B after impact is v^1 at β to barrier		Allow any specified value of v
			$\beta = 90 - \alpha$	B1	
			either		
			Use $\tan \beta = e \tan \alpha$	M1	May be quoted without proof
			$\tan \beta = \dfrac{1}{3} \tan \alpha$	A1	
			or		
			$v \cos \alpha = v^1 \cos \beta$ and $v^1 \sin \beta = \dfrac{1}{3} v \sin \alpha$	M1	Award for either statement seen
			$\tan \beta = \dfrac{1}{3} \tan \alpha$	A1	
			or		
			$v_x = u_x$ and $v_y = \dfrac{1}{3} u_y$	M1	Both statements needed or seen on diagram
			$\tan \beta = \dfrac{v_y}{v_x} = \dfrac{1}{3} \dfrac{u_y}{u_x} = \dfrac{1}{3} \tan \alpha$	A1	
			then		
			so $\dfrac{1}{\tan \alpha} = \dfrac{1}{3} \tan \alpha$	A1	Any form, in terms of $\tan \alpha$ only
			so $\tan^2 \alpha = 3$ and $\tan \alpha = \sqrt{3}$ (+ve root) and $\alpha = 60°$	A1	cao
				[5]	

Question			Answer	Marks	Guidance	
2		(i)	$0 - \dfrac{1}{2} \times 5 \times 10^2 = -5 \times 10 \times 2 - \text{WD}$	M1	M1 use of W-E equation with WD, KE and GPE. Allow sign errors	
				A1	Any form. Allow sign error in WD term only	
			So WD = 150. Work done is 150 J	A1	-150 gets $2/3$ (Allow 152 from $g = 9.8$)	
				[3]		
		(ii)	Say Q moves d up ramp and friction is F WD = Fd	M1	used	
			$d = \dfrac{2}{\sin \alpha}$ so WD = $\dfrac{2F}{\sin \alpha}$	A1		
			(Since sliding) $F = \mu R$	M1	used	
			Resolving perp to ramp $R = 5 \times 10 \cos \alpha$	B1	$5g \cos \alpha$	
			so $F = 50 \times \dfrac{5}{8} \cos \alpha = \dfrac{125}{4} \cos \alpha$			
			Hence WD is $\dfrac{2}{\sin \alpha} \times \dfrac{125}{4} \cos \alpha = \dfrac{125}{2 \tan \alpha}$	A1	AG Properly shown	
			so $150 = \dfrac{125}{2 \tan \alpha}$	M1	Equate WD to 150. FT(i)	
			and $\tan \alpha = \dfrac{125}{2 \times 150} = \dfrac{5}{12}$ (0.417 to 3 s. f.)	A1	cao aef NOT implied by 22.6	
				[7]		
		(iii)	**either** Suppose α is greater, $\cos \alpha$ is less so $F = 50 \times \dfrac{5}{8} \cos \alpha$ is less also $\sin \alpha$ is greater so $d = \dfrac{2}{\sin \alpha}$ is less Hence $Fd = $ WD is less. GPE the same so Q	B1	Award for either F or d stated to be less OR still moving at D	

Question			Answer	Marks	Guidance	
			is still moving	B1	Award for both F and d stated to be less AND still moving at D	
			or From (ii) use WD $= \dfrac{125}{2\tan\alpha}$ If α is greater, $\tan\alpha$ is greater Hence WD is less. GPE the same so Q is still moving	B1 B1 **[2]**	Award for WD stated to be less OR still moving at D Award for WD stated to be less, with reference to established result, AND still moving at D	
		(iv)	Using N2L $\dfrac{50}{v} - F = 5a$	M1 B1 A1 **[3]**	Use of N2L All terms present ($D - F = 5a$ is sufficient) Use of *Power = Driving force × speed* Any form	
		(v)	Put $a = 0$ and $v = 4$, $F = 12.5$.	B1 **[1]**		
		(vi)	a and v are the only things in the equation that can change so they are both constant or both vary If $a \neq 0$, v must vary so a must vary. Hence if a is constant it must be zero (giving constant v) *suvat* equations only valid for constant accn	B1 B1 B1 **[3]**	F is constant OR a and v are the only variables Must be convincing	

Question			Answer	Marks	Guidance	
3	**(a)**				In this part, award the best possible mark for marks 3 to 8.	
					M1A1 for a correct resolution and force (even if not the first one given)	
					M1F1 for second resolution and force	
					M1 for third resolution	
					A1 All forces correct and all T/C correct	
			$\sin\alpha = 0.6;\ \cos\alpha = 0.8$	B1	Either of these	
			Horizontal equilibrium of whole framework:			
			$80\cos\alpha - X = 0$ so $X = 64$	B1		
			Take the internal forces to be +ve in tension		Any convention may be used	
			At K↑ $80\sin\alpha + T_{KL}\cos\alpha = 0$	M1	All relevant forces only, resolved if necessary, allow sign errors and cos/sin mix	
			so $T_{KL} = -60$ a force in KL of 60 N (C)	A1		
			either			
			At K resolve parallel to JL:	M1	All relevant forces only, resolved if necessary, allow sign errors and cos/sin mix	
			$80 - T_{JK}\cos\alpha = 0$			
			or			
			At K→ $80\cos\alpha - T_{KL}\sin\alpha - T_{JK} = 0$	M1	All relevant forces only, resolved if necessary, allow sign errors and cos/sin mix	
			so $T_{JK} = 100$ a force in JK of 100 N (T)	F1	FT their value for KL	
			either			
			At J→ $T_{JK} + T_{LJ}\cos\alpha - X = 0$	M1		
			so $T_{LJ} = -45$ a force in LJ of 45 N (C)	A1	All values correct and all T/C correct	
			or			
			At L→ $T_{KL}\sin\alpha - T_{LJ}\cos\alpha = 0$	M1		
			so $T_{LJ} = -45$ a force in LJ of 45 N (C)	A1	All values correct and all T/C correct	
				[8]		

Question			Answer	Marks	Guidance	
	(b)		Take: weight of ladder as W N; angle of ladder with horiz as α. Forces: at A, R N ↑, F N ←; at C, S N perp to AB			
			$\tan\alpha = \dfrac{3}{1.25} = \dfrac{12}{5}$; $\sin\alpha = \dfrac{12}{13}$; $\cos\alpha = \dfrac{5}{13}$	B1	Award for any of these seen. oe for different angle chosen (Award for 67.4)	
			ac moments about A $2.5W\cos\alpha - 3.25S = 0$ (3) $S = \dfrac{50W}{169}$	M1 B1 A1	Moments with all terms present The distance AC is 3.25 (may be implied) Shown: A0 if 67.4 stated with no evidence of 12/5 o.e.	
			In equilib; consider vert and horiz cpts ↑ $R + S\cos\alpha - W = 0$ (1) ← $F - S\sin\alpha = 0$ (2)	M1 A1 M1 A1	Award for either. Allow cos/sin mix, allow sign errors oe (R = 0.886W or 1947/2197 W) Second resolution. Allow cos/sin mix, allow sign errors oe (F = 0.273W or 600/2197 W)	
			We have $\mu \geq \dfrac{F}{R}$ Substitute for $\sin\alpha$ and $\cos\alpha$ and for F and R from (1) and (2). Eliminate S.	M1	Used and attempt to get all in terms of W (Accept '='at this stage)	
			so $\mu \geq \dfrac{\dfrac{50W}{169} \times \dfrac{12}{13}}{W - \dfrac{50W}{169} \times \dfrac{5}{13}}$	F1	Correct ft, dependent on all of previous 3 M marks, all in terms of W	
			so $\mu \geq \dfrac{600W}{(169 \times 13 - 250)W} = \dfrac{200}{649}$	A1	This value is 0.3081664…: 600/1947	

Question			Answer	Marks	Guidance	
			Hence $\mu \geq 0.308$ (3 s. f.)	A1	cao as final answer. Must have > as well as =.	
				[12]		

Question			Answer	Marks	Guidance	
4	(a)	(i)	$(2+2+3+5)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=2\begin{pmatrix}3\\2\end{pmatrix}+2\begin{pmatrix}3\\0\end{pmatrix}+3\begin{pmatrix}2\\4\end{pmatrix}+5\begin{pmatrix}6\\4\end{pmatrix}$	M1	Complete method for CoM Using 24 instead of 2 for mass of lamina is NOT a MR: award M1 max. Omitting mass of lamina, M1max. Allow one error	
			so $12\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=\begin{pmatrix}48\\36\end{pmatrix}$			
			and the CoM is at (4, 3)	A1	One coordinate cao	
				A1	Other coordinate cao	
				[3]		
	(ii)		Require			
			$\begin{pmatrix}48\\36\end{pmatrix}+m\begin{pmatrix}X\\0\end{pmatrix}=(12+m)\begin{pmatrix}p\\\dfrac{2p}{3}\end{pmatrix}$ for some p	M1	General method for attempt to get new CoM on OB	
				B1	Dealing with finding a point on the line OB: using $y=\dfrac{2}{3}x$ in some way	
			Hence $48+mX=(12+m)p$			
			and $36=(12+m)\dfrac{2p}{3}$	M1	Consider the components and attempt to eliminate, or equivalent	
			Eliminate $(12+m)p$ to get			
			$36=\dfrac{2}{3}(48+mX)$ so $mX=6$	A1	Convincingly done (must follow from correct (48, 36))	
				[4]		
	(b)	(i)	Let the line density of the wire be λ.		Do not penalise for stating $\lambda=1$ or assuming this without comment	
			$\bar{y}=0$, by considering symmetry	B1	Need a statement including symmetry (or calculation for this coordinate)	
			DE has mass $kr\lambda$, CoM at (0, 0). Semi-circle has mass $\pi r\lambda$, CoM at $\left(\dfrac{2r}{\pi},\ 0\right)$.	B1	Both masses correct (but see above, accept kr and πr)	

Question			Answer	Marks	Guidance	
			$\left(kr\lambda + \pi r\lambda\right)\bar{x} = kr\lambda \times 0 + \pi r\lambda \times \dfrac{2r}{\pi}$	M1	Method for CoM. Accept 1st term on RHS not present	
			so $\left(k + \pi\right)\bar{x} = 2r$ and $\bar{x} = \dfrac{2r}{k+\pi}$	A1	Must be convincing	
				[4]		
		(ii)	Suppose the weight of P is W and CoM at G **Method A** Take c.w moments about G		Condone use of length of wire or of mass instead of weight in correct ratios	
			$T_{\mathrm{E}} \times \dfrac{2r}{\pi+k} - T_{\mathrm{B}} \times \left(r - \dfrac{2r}{\pi+k}\right) = 0$	M2	Use of moments with all appropriate forces	
				A1	A correct moments equation	
			Substitute $T_{\mathrm{E}} = 2T_{\mathrm{B}}$ giving	M1	Substitute and attempt elimination	
			$\dfrac{4r}{\pi+k} = r - \dfrac{2r}{\pi+k}$	A1	o.e.	
			so $4r = r(\pi+k) - 2r$ so $k = 6 - \pi$ (2.86 to 3 s. f.)	A1	cao	
			Method B			
			a.c moments about O: $W \times \dfrac{2r}{\pi+k} - T_{\mathrm{B}} \times r = 0$	M1 A1	Use of moments with all appropriate forces A correct moments equation. (Allow $\pi r + kr$ as W)	
			so $T_{\mathrm{B}} = \dfrac{2W}{\pi+k}$			
			Using vertical components: $T_{\mathrm{B}} + T_{\mathrm{E}} = W$	M1	Or take moments about a second point, for example B	
			so $T_{\mathrm{E}} = W - \dfrac{2W}{\pi+k}$			

Question	Answer	Marks	Guidance
	$= \dfrac{W(\pi + k - 2)}{\pi + k}$		
	$\dfrac{T_E}{T_B} = 2 = \dfrac{\dfrac{W(\pi + k - 2)}{\pi + k}}{\dfrac{2W}{\pi + k}}$	M1	Using their expressions for tensions
	so $4 = \pi + k - 2$	A1	Some simplification seen
	so $k = 6 - \pi$ (2.86 to 3 s. f.)	A1	cao
	Method C		
	$T_E{:}T_B = GB{:}OG$	M2	
	$GB = r - \dfrac{2r}{\pi + k} = \left(\dfrac{r(\pi + k - 2)}{\pi + k} \right)$	A1	
	$\dfrac{T_E}{T_B} = 2 = \dfrac{\dfrac{W(\pi + k - 2)}{\pi + k}}{\dfrac{2W}{\pi + k}}$	M1	Using their expressions for lengths
	so $4 = \pi + k - 2$	A1	Some simplification seen
	so $k = 6 - \pi$ (2.86 to 3 s. f.)	A1	cao
		[6]	