OCR
Oxford Cambridge and RSA

GCE

Mathematics

Unit **4733**: Probability and Statistics 2

Advanced GCE

Mark Scheme for June 2017

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Subject-specific Marking Instructions for GCE Mathematics (OCR) Statistics strand

Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting probabilities from Normal tables, we generally expect *some* evidence of interpolation and so quotation to 4 decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice, especially if the calculated value of a test statistic is nowhere near any of these values. Sensible discretion *must* be exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem demands it) but only 2 are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, answers which are *grossly* over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example, insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to answers that are given as final stages of calculations; intermediate working should usually be carried out, and quoted, to a greater degree of accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/or difficulty of the question being considerably changed, it is likely that all the marks for that question, or section of the question, will be lost. However, misreads are often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but at least some accuracy ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even when the object and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps as a quoted value of a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so

slight an effect as to be almost unnoticeable in the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be more than 2. Commonly in sections of questions where there is a numerical answer either at the end of the section or to be obtained and commented on (eg the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark must be withheld even if all method marks have been earned. It will also often be the case that such a mark is implicitly "cao" even if not explicitly designated as such.

On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-through of the candidate's value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the candidate may exhibit knowledge of how to compare it with a critical value and draw conclusions. Such " fresh starts" are not affected by any earlier misreads.

A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. Such misreads are more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they do not, they should be treated as far as possible in the same way as numerical misreads, *mutatis mutandis*. This also applied to misreads of text, which are fairly rare but can cause major problems in fair marking.

The situation regarding any particular cases that arise while you are marking for which you feel you need detailed guidance should be discussed with your Team Leader.

Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

1	(i)	Biased against those not at the parents' evening	B1 **1**	Reason for being biased or unrepresentative, needs more than "not all will be at the meeting", e.g. "not all will return the questionnaire" or "those at the meeting may have different opinions"	"Biased" can be implied by the reason Not *just* "not random" or "not representative", but allow "self-selecting". Ignore irrelevancies (e.g. "small sample", but withhold if definitely wrong comment seen
	(ii)	Obtain list of parents/pupils & number it 1 to *n* Select using random numbers, ignoring repeats/numbers outside range	B1 B1 **2**	Number (a list of) parents (sequentially) (statements in brackets can be implied) Mention use of RNs, as *only* method, *and* either "ignore repeats" or "ignore outside range" (allow "use RNs in range")	SC: Allocate *random* numbers: max B1 unless *sorted* *Not* "select numbers randomly" *Not* hat/lottery machine [RNs required by question] Allow systematic provided random start
2		$\frac{70-\mu}{\sigma}=0.842$; $\frac{81-\mu}{\sigma}=1.282$ $11 = 0.44\sigma$ [$\sigma = 25$] $\mu = 48.93$ $81 - \mu = \mu - a$ $a = 16.9$	M1* A1 B1 dep*M1 A1 M1 A1 **7**	Stand'ise once & equate to Φ^{-1}, allow sign/√/cc errors LHS both correct, signs consistent on both sides Both *z* values, ∈ [0.841, 0.842] and [1.281, 1.282] Solve to get μ or σ Either, $\mu \in$ [48.9, 49(.0)], $\sigma \in$ [25, 25.1], www Equation for *a*, correct signs [may involve 1.282] *a* in range [16.8, 17], www	"P(> 81) = P(≥ 80)" etc or "1 – 1.282" is M1A0 Can get M1A1 even if *z* wrong provided they <u>are</u> *z* Can award B1 even if signs are wrong e.g. $\mu - a = 1.282\sigma$ [*a* needs to be less than their μ]
3		H_0: $\lambda = 13$, H_1: $\lambda > 13$ α: P(≥ 23) = 1 – 0.9924 = 0.0076 < 0.01	B2 M1 A1 A1	One error, B1, but *x*, *t* etc or 23: B0 Find P(≥ 23, > 23, = 23, < 23, ≤ 23) from Po(13) P(≥ 23) = 0.0076 ONLY (but see SC below] Explicit comparison with 0.01 SC: 0.9924 > 0.99: M1A2 and can get last M1A1	H_0: $\mu = 13$, H_1: $\mu > 13$ gets full marks 0.9924, 0.996, 0.0040, 0.0036: M1A0A0M0A0 SC: "P(≥ 23) = 1 – P(≤ 23)": M1A0A1M1A1ft SC: 1 – 0.9970 = 0.003 or 1 – 0.9833 = 0.0167 *[from λ = 12 or 14]*: A0 but can get all other marks
		β: CR ≥ 23 Probability 0.0076 23 in CR	B1dep* dep*B1 dep*B1	Must be clearly stated [*not* just "CV = 23"] Must be seen, but allow 0.9924 [e.g. on diagram] Must be stated explicitly	SC: If *mixture* of methods: maximum (B2)M1 [or (B2)B1], M1A1, max 5/7 *Second and third B1 are independent of each other* SC: CR ≥ 24, *p* = 0.0093 B1* (for both) 23 not in CR, DNR B1dep, M1A1 *or* CR ≥ 22, *p* = 0.0061 B1* (for both) 23 in CR, reject B1dep, M1A1 *[from λ = 12 or 14]* *either could get 6/7*
		Reject H_0. There is significant evidence that the new team makes more mistakes.	M1 A1 **7**	First conclusion consistent, needs correct method Contextualised, acknowledge uncertainty	

4	(i)	$\hat{\mu} = \bar{x} = 6.18$ $\hat{\sigma}^2 = \frac{36}{35}\left(\frac{1380.5264}{36} - 6.18^2\right)$ $= 0.16$ H_0: $\mu = 6.3$, H_1: $\mu \neq 6.3$ α: $z = \frac{6.18-6.3}{\sqrt{0.16/36}} = -1.8$, $p = 0.0359$ $-1.8 > -1.96$ or $0.0359 > 0.025$	B1 M1 M1 A1 B2 M1 A1 A1	6.18 seen somewhere Correct formula for biased estimate Multiply by 36/(36 – 1) 0.1556: M1M0A0. Allow e.g. 5.6/35 One error, B1, but x, $\bar{x}$, t : B0 Standardise, 36 needed (if omitted, no more marks in (i) 1.8 or –1.8 or a.r.t. 0.0359 Compare $-z$ with –1.96 or z with 1.96 or p with 0.025, like-with-like	Single formula: M2 if right, M1 if wrong but with 35 divisor *somewhere* H_0: $\lambda = 6.3$, H_1: $\lambda \neq 6.3$: B1 u rather than μ: B1B0 if unquestionably u, else BOD Allow 0.9641 *only* if compared with 0.95 or 0.975 Wrong or no notation (e.g. "cdfnorm"): full marks if right, M0A0A0 (M1A1) if numbers wrong in any way
		β: CV $6.3 - 1.96\sqrt{\frac{0.16}{36}} = 6.1693$ $6.18 > 6.1693$	M1 A1 A1	$6.3 - z\sqrt{(\sigma^2/36)}$, allow √ errors, cc, ± $z = 1.96$ Compare $\bar{x}$ with 6.17 (or with 6.19 from 1-tail)	$6.18 + z\sqrt{(\sigma^2/36)}$: M1 and no further marks in (i) 6.17 (and no working) can imply mark for 1.96 SC 1-tail: $6.18 > 6.19$, reject H_0, etc: M1A0A1, M1A1
		Do not reject H_0. Insufficient evidence that pH of paper is not 6.3	M1 A1ft **11**	Requires essentially correct method, 36 divisor, like-with-like, hypotheses involving 6.3 Contextualised, acknowledge uncertainty. Allow "insufficient evidence that pH hasn't changed"	Withhold A1 if no context or too assertive, e.g. "evidence that pH of paper is 6.3"
	(ii)	In comparing z with z_{crit}	B1 **1**	Or in using 1.96 for CV, etc, or "in assuming that the sample (mean) is normally distributed" (must answer "where?", mustn't leave it vague as to whether it's X or $\bar{X}$)	No extra answers. "Calculating variance" or "dividing by $\sqrt{n}$" : B0. Not "because the population is not known to be normally distributed". But allow if OK *and also* explained why it can be used.
5	(i)	B(74, 1/37) ≈ Po(2) P(≥ 4) = 1 – P(≤ 3) = 1 – 0.8571 = 0.1429 $n > 50$, (np =) $2 < 5$	M1 M1 A1 B1 **4**	Po(2) stated or implied RH tail of their Poisson Answer, a.r.t. 0.143 Both conditions, dep on Poisson Allow "n large, p small" or "n large, $np < 5$". FT on their $74\times(1/37)$ if less than 5	If formula used, must be correct formula, and > 1 term 0.14037 (from exact binomial): 0 np condition needs their 2 seen *somewhere* If numbers used, must be compared with 50 and 5 Extra or wrong conditions (e.g. nq): B0 *If MR leading to np > 5, please consult TL*
	(ii)	B(148, 18/37) ≈ N(72, 36.973) $72 + 0.5 + 2.807\times\sqrt{36.973}$ = 89.57 Hence $N_{min} = 90$	M1 A1 M1 B1 A1 A1 **6**	N(np, …) attempted Both parameters correct, allow √ errors [$= \frac{1368}{37}$] $72 + z\sigma$, allow σ^2 and/or no cc, do not allow $\sqrt{n}$ divisor $z = 2.807$ or 2.808 or 2.809 or a.r.t. 2.81, allow –ve [can be implied by 89.6 or 89.1 or 88.9] 89.6 or 89.1 seen or implied 90 only, www (although cc can be omitted if rounded up)	• Must see evidence of appropriate approximation • No working: 0 (approximation required) • 0.00196 implies exact binomial: 0 • BUT $\sqrt{37}$ may be $\sqrt{36.97}$, not $\sqrt{n}$ Ignore inequalities until final mark Allow from 89.07 and checked using exact binomial

6	(i)	Articles must be received independently of one another and at constant average rate	B1 B1 **2**	At least one must be contextualised for *any* marks Independent stated, allow "probability independent" Allow "uniform" rate but not "constant" rate *Not* "probability is constant"	If extras, e.g. "singly" or "randomly", then max 1 Allow "receipt of one doesn't affect receipt of another" Any implication of regularity: *can't* get second B mark
	(ii)	Po(4.8), left-hand tail P(< 5) = 0.4763	M1 A1 **2**	Po(2×2.4), stated or implied, e.g. by 0.6510	0.4946 or 0.4582: M1A0
	(iii)	$e^{-2.4}\frac{2.4^r}{r!} = 2.5\times e^{-2.4}\frac{2.4^{r+1}}{(r+1)!}$ $r + 1 = 2.5$ 2.4 $r = 5$	M1* A1 *depM1 A1 **4**	One correct probability from formula Correct equation, allow from $\lambda = 4.8$ [= MR] Simplify exp and ! correctly to linear equation in r $r = 5$ only, www	SC: T&I, or tables or calculator: 0 $r + 1$!: allow if used as if it were $(r + 1)$! Wrong use of logs: M0A0
	(iv)	Po(120) ≈ N(120, 120) $1-\Phi\left(\frac{139.5-120}{\sqrt{120}}\right)$ $= 1 - \Phi(1.78) = 0.0375$	M1 A1 M1 A1 A1 **5**	Normal stated or implied, mean 50×2.4 Both parameters correct, allow √ errors Standardise, allow no/wrong cc and/or √ errors, can be implied by correct answer Both cc and correct √ Answer, anything rounding to 0.0375	If answer wrong, do not give M1 unless correct *notation* used (i.e., calculator notation such as "cdfnorm" does not qualify for M1 even if answer is recognisable)
7	(i)	$\int_0^4 \frac{1}{64}x^2(16-x^2)\mathrm{d}x$ = $\left[\frac{1}{64}\left(\frac{1}{3}.16x^3-\frac{1}{5}x^5\right)\right]_0^4$ *or* $\left[\frac{1}{12}x^3-\frac{1}{320}x^5\right]_0^4$ $\frac{32}{15}$ or 2.133…	M1 B1 A1 **3**	Attempt to integrate xf(x), correct limits *somewhere* Correct indefinite integral, aef Answer, exact or anything rounding to 2.13	Allow numerical, but if algebraic, powers must increase to score M1
	(ii)	(a) $\int_0^q \frac{1}{64}x(16-x^2)\mathrm{d}x = \frac{3}{4}$ $\frac{1}{64}\left(8q^2-\frac{1}{4}q^4\right)=\frac{3}{4}$ *or* $\frac{1}{8}q^2-\frac{1}{256}q^4=\frac{3}{4}$ $q^4 - 32q^2 + 192 = 0$	M1 A1 A1 **3**	Correct integral, limits 0, q Correct unsimplified equation Correctly obtain given equation, www	Allow limits q, 4 if equated to ¼, not otherwise Withhold if "simplified", e.g. to $-32q^6 + 192 = 0$
		(b) $q^2 = 8$ or 24 $q = \pm\sqrt{8}$ or $\pm\sqrt{24}$ $q = \sqrt{8} = 2\sqrt{2}$	M1 A1 A1 **3**	Solve for q^2 Acknowledge other answers (or explicitly reject) $q = \sqrt{8}$ or $2\sqrt{2}$ as sole answer, allow 2.83 as sole final answer *only if* exact also seen somewhere	Numerical only: give M1 for 2.83 (2.828…) seen $\sqrt{8}$, or not ±, and no other comment: M1A0A1

8	(i)	$nq = 3 < 5$, or p not close to ½ and n not large enough	B1 **1**	www. 3 and 5 must be seen if inequality used (No need to mention Poisson – ignore any mention)	Withhold if extra wrong statements seen but ignore irrelevant statements (e.g. np). Do *not* allow $npq = 2.85$
	(ii)	$P(X = 60) = 0.95^{60} = 0.046$ < 0.05 $P(X \geq 59) = 0.046 + 0.1455$ [= 0.19155] > 0.05	B1dep* dep*B1 B1dep† dep†B1 **4**	In range [0.0460, 0.0461], *or* [0.9539, 0.954] Correct tail explicitly compared In range [0.191, 0.192], *or* [0.808, 0.809] Correct tail explicitly compared (no final conclusion needed)	0.145 or 0.855 qualifies for these two marks SC: Po(3): $P(X = 60) = 0.0498$ B1dep* < 0.05, dep*B1 $P(X \geq 59) = 0.1991$ B1dep† > 0.05: dep†B1
	(iii)	$p = 0.95$ 0.046	B1 B1ft **2**	Question requires this to be stated FT on their 0.046 from (ii)	*NOT* 0.05. NB: if Po(3) used in (ii), 0.0498 gets B1
	(iv)	$P(\leq 59) < 0.6$ $1 - p^{60} < 0.6$ $p^{60} > 0.4$ $0.985 < p \leq 1$	M1 A1 A1 A1 **4**	Can be implied Range required, allow any combination of </≤ Withhold if more than 5 sf seen	$p^{60} > 0.6$ or $p^{60} < 0.6$: can get M1 if $P(\leq 59) < 0.6$ stated explicitly, otherwise 0 SC: T&I or tables: $p > 0.985$ or better, B4, else 0 Allow $p > 0.958$ SC: $60 - R \sim$ Po(λ): $1 - P(\leq 0) < 0.6$ M1 $e^{-\lambda} > 0.4$ A1 ($\lambda < 0.916$) $60(1 - p) < 0.916$ M1, needs M1A1 $p > 0.985$ A1

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