Oxford Cambridge and RSA

Friday 23 June 2017 – Morning

A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 **(i)** Express $\dfrac{5-x}{(2-x)(1+x)}$ in partial fractions. **[3]**

(ii) Hence or otherwise find the first 3 terms of the binomial expansion of $\dfrac{5-x}{(2-x)(1+x)}$ in ascending powers of x. **[5]**

2 The equation of a line is $\mathbf{r} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and the equation of a plane is $3x + 4y - z = 17$.

(i) Find the coordinates of the point of intersection of the line and the plane. **[4]**

(ii) Find the acute angle between the line and the normal to the plane. **[4]**

3 Fig. 3 shows the curve $y = \sqrt{1+e^{2x}}$.

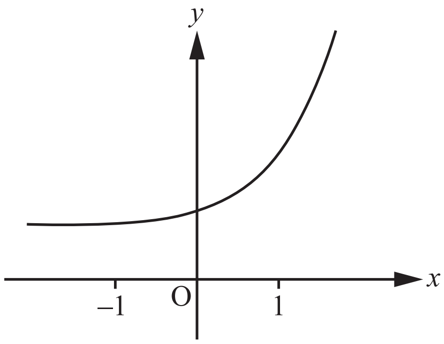

Fig. 3

The value of $\int_{-1}^{1} \sqrt{1+e^{2x}}\,dx$ is to be estimated using the trapezium rule. T_2 and T_4 are the estimates obtained from the trapezium rule using 2 strips and 4 strips respectively.

(i) Explain whether T_4 is greater or less than T_2. **[2]**

(ii) Evaluate T_4, giving your answer to 3 significant figures. **[4]**

4 Vectors **u** and **v** are given by $\mathbf{u} = \mathbf{i} - 7\mathbf{j} - 2\mathbf{k}$ and $\mathbf{v} = a\mathbf{i} + b\mathbf{j} + 5\mathbf{k}$, where a and b are constants.

Find a and b given that the magnitude of **v** is $\sqrt{27}$ and that **u** and **v** are perpendicular. **[6]**

5 Solve the equation $4\tan\theta\tan 2\theta = 1$, for $0° < \theta < 180°$. **[4]**

6 The number of bacteria in a population at time t is denoted by P. The rate of increase of P is proportional to the square root of P.

(i) Write down a differential equation relating P, the time t, and a constant of proportionality k. **[1]**

(ii) Verify that $P = (A + Bt)^2$, where A and B are constants, satisfies the differential equation, and find k in terms of B. **[3]**

Section B (36 marks)

7 The curve shown in Fig. 7 passes through the origin and satisfies the differential equation

$$\frac{dy}{dx} = \frac{9x}{4(y+3)}.$$

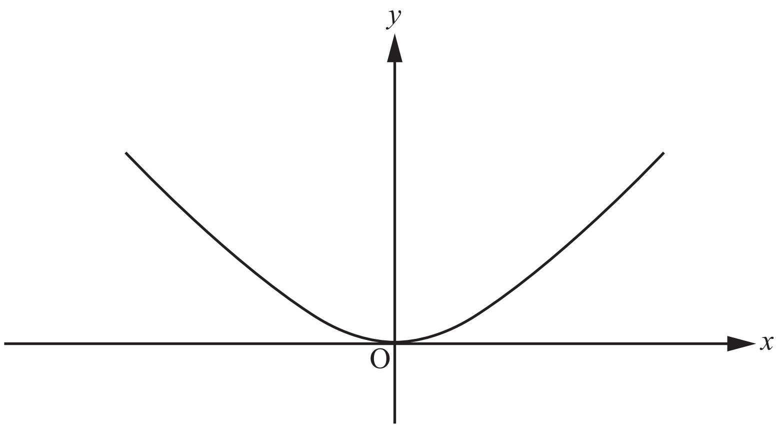

Fig. 7

(i) Show by integration that the equation of the curve is $9x^2 - 4y^2 - 24y = 0$. **[5]**

The finite region bounded by the curve and the line $y = 2$ is rotated through 180° about the y-axis.

(ii) Find the volume of the solid of revolution generated, giving your answer as an exact multiple of π. **[4]**

(iii) Use the substitutions $x = 2\tan\theta$ and $y = 3(\sec\theta - 1)$

(A) to verify that $9x^2 - 4y^2 - 24y = 0$,

(B) to show that $\dfrac{9x}{4(y+3)}$ can be expressed as $k\sin\theta$, where k is a constant to be found.

Hence find the exact gradient of the curve at the point with x-coordinate 2. **[9]**

8 Fig. 8 shows the curve with parametric equations

$$x = \cos 2\theta,\ y = \cos\theta + 2\sin\theta,\ \text{for } -\pi < \theta \leqslant \pi.$$

The curve intersects the x-axis at A, and the points B and C have maximum x- and y-coordinates respectively.

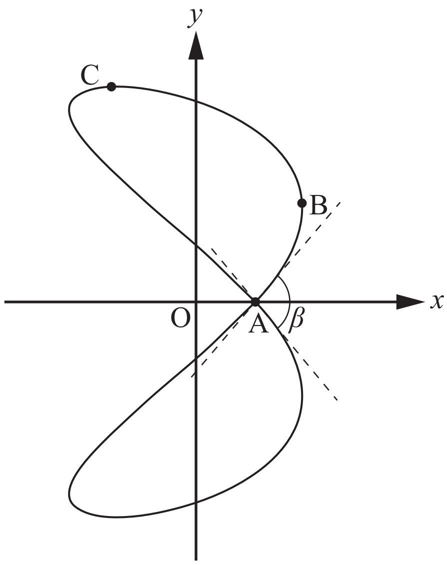

Fig. 8

(i) Find the value of θ corresponding to the point B. Hence find the coordinates of the point B. **[3]**

(ii) Express $\cos\theta + 2\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. **[5]**

(iii) Hence find the coordinates of the points A and C. **[5]**

The angle β is the angle between the tangents to the curve at A.

(iv) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ. Hence, assuming the scales of the x- and y-axes are equal, find β, giving the answer in radians correct to 2 decimal places. [You may assume the curve is symmetrical about the x-axis.] **[5]**

END OF QUESTION PAPER