OCR
Oxford Cambridge and RSA

Tuesday 20 June 2017 – Afternoon

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Differentiate $\dfrac{1}{(5-2x^3)^2}$. **[3]**

2 The function $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = |x|$, for $-1 \leqslant x \leqslant 1$.

Sketch the graph of $y = \mathrm{g}(x)$, where $\mathrm{g}(x) = 2 - 2\mathrm{f}(x)$. **[3]**

3 Functions $\mathrm{f}(x)$ and $\mathrm{g}(x)$, each defined for $-1 < x < 1$, are given by $\mathrm{f}(x) = \ln(1-x)$ and $\mathrm{g}(x) = x^2$.

(i) Find $\mathrm{f}^{-1}(x)$ and state its domain and range. **[4]**

(ii) Show that $\mathrm{f}(x) + \mathrm{f}(-x) = \mathrm{fg}(x)$. **[3]**

4 A curve has equation $3x^{\frac{2}{3}} + 2y^{\frac{1}{3}} = 7$.

(i) By differentiating implicitly, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. **[3]**

(ii) Hence find the gradient of the curve at the point with coordinates (1, 8). **[2]**

5 A liquid is being heated. At time t minutes after heating starts, its temperature, $\theta\,°\mathrm{C}$, is modelled by the equation

$$\theta = 10.5 + 69.5(1 - \mathrm{e}^{-kt}),$$

where k is a positive constant. The boiling point of the liquid is the value approached by θ as t tends to infinity.

(i) Write down the initial temperature and the boiling point of the liquid. **[2]**

(ii) After being heated for one minute, the liquid has a temperature of 30 °C. Find k. **[3]**

(iii) Find how long it takes from the start of the heating until the temperature is within 1 °C of the boiling point. Give your answer to the nearest minute. **[3]**

6 You are given that the sum of the interior angles of a polygon with n sides is $180(n - 2)°$. Using this result, or otherwise, prove that the interior angle of a regular polygon cannot be 155°. **[3]**

7 The equation of a curve is $y = \arcsin\frac{1}{2}x$.

(i) Express each of x and $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y. **[2]**

(ii) A point is moving on the curve, and has coordinates (x, y) at time t. When $x = 1$, the value of $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ is 2.

Find the exact value of $\dfrac{\mathrm{d}y}{\mathrm{d}t}$ at this instant. **[5]**

Section B (36 marks)

8 Fig. 8 shows part of the curve $y = \dfrac{\cos x}{2 - \sin x}$. The curve intersects the x- and y-axes at A and C respectively, and has a turning point at B.

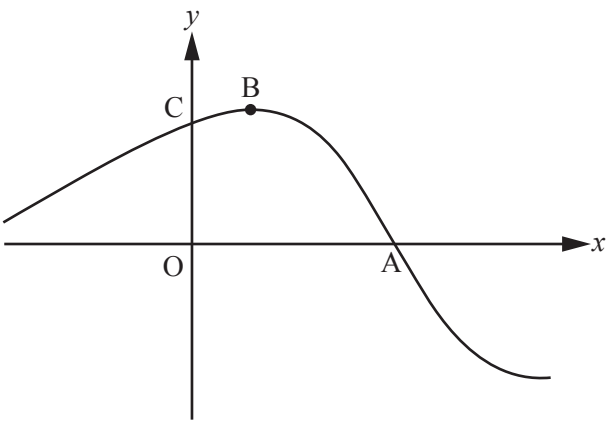

Fig. 8

(i) Write down the coordinates of A and C. **[2]**

(ii) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and the exact coordinates of B. **[7]**

(iii) (*A*) Using integration by substitution, or otherwise, find the exact area of the region enclosed by the curve, the y-axis and the positive x-axis. **[4]**

(*B*) The line $x = k$ divides this region into two parts of equal area. Show that $k = \arcsin\left(2 - \sqrt{2}\right)$. **[5]**

9 A curve has equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = x^3 \mathrm{e}^{-x^2}$.

(i) Show that $\mathrm{f}(x)$ is an odd function, and interpret this result in terms of the graph of the curve $y = \mathrm{f}(x)$. **[3]**

(ii) Find the coordinates of the stationary points of the curve. Give answers correct to 2 decimal places where appropriate. **[7]**

(iii) Sketch the curve for $-2 \leqslant x \leqslant 2$. **[2]**

(iv) (*A*) Show, using the substitution $t = x^2$, that $\int \mathrm{f}(x)\,\mathrm{d}x$ may be expressed as $\int kt\,\mathrm{e}^{-t}\,\mathrm{d}t$, where k is a constant to be determined. **[2]**

(*B*) Hence find the exact area of the region enclosed by the curve $y = \mathrm{f}(x)$, the positive x-axis and the line $x = 2$. **[4]**

END OF QUESTION PAPER