Oxford Cambridge and RSA

GCE

Mathematics

Advanced GCE A2 7890 – 2

Advanced Subsidiary GCE AS 3890 – 2

OCR Report to Centres June 2017

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About this Examiner Report to Centres

This report on the 2017 Summer assessments aims to highlight:

- areas where students were more successful
- main areas where students may need additional support and some reflection
- points of advice for future examinations

It is intended to be constructive and informative and to promote better understanding of the specification content, of the operation of the scheme of assessment and of the application of assessment criteria.

Reports should be read in conjunction with the published question papers and mark schemes for the examination.

The report also includes:

- An invitation to get involved in Cambridge Assessment's research into **how current reforms are affecting schools and colleges**

- Links to important documents such as **grade boundaries**

- A reminder of our **post-results services** including Enquiries About Results

- **Further support that you can expect from OCR**, such as our Active Results service and CPD programme

- A link to our handy Teacher Guide on **Supporting the move to linear assessment** to support you with the ongoing transition

Understanding how current reforms are affecting schools and colleges

Researchers at Cambridge Assessment[1] are undertaking a research study to better understand how the current reforms to AS and A levels are affecting schools and colleges.
If you are a Head of Department (including deputy and acting Heads), then we would be very grateful if you would take part in this research by completing their survey. If you have already completed the survey this spring/summer then you do not need to complete it again.
The questionnaire will take approximately 15 minutes and all responses will be anonymous.
To take part, please click on this link: https://www.surveymonkey.co.uk/r/KP96LWB

Grade boundaries

Grade boundaries for this, and all other assessments, can be found on Interchange. For more information on the publication of grade boundaries please see the OCR website.

Enquiry About Results

If any of your students' results are not as expected, you may wish to consider one of our Enquiry About Results services. For full information about the options available visit the OCR website. If university places are reliant on the results you are making an enquiry about you may wish to consider the priority 2 service which has an earlier deadline to ensure your enquires are processed in time for university applications.

Further support from OCR

activeresults

Active Results offers a unique perspective on results data and greater opportunities to understand students' performance.
It allows you to:

- Review reports on the **performance of individual candidates**, cohorts of students and whole centres
- **Analyse results** at question and/or topic level
- **Compare your centre** with OCR national averages or similar OCR centres.
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http://www.ocr.org.uk/getting-started-with-active-results

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https://www.cpdhub.ocr.org.uk

[1] Cambridge Assessment is a not-for-profit non-teaching department of the University of Cambridge, and the parent organisation of OCR, Cambridge International Examinations and Cambridge English Language Assessment.

Supporting the move to linear assessment

September 2017 sees the introduction of linear assessment for AS and A level mathematics and further mathematics. To help you navigate the changes and to support you with areas of difficulty, download our helpful Teacher guide: http://www.ocr.org.uk/Images/338121-moving-from-modular-to-linear-qualifications-teachers-guide.pdf

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OCR REPORT TO CENTRES

4721 Core Mathematics 1

General Comments

The vast majority of candidates were, as ever, very well prepared for this paper, although a significant number seemed unfamiliar with the use of answer books. This caused problems for markers in finding the candidates' solutions and it would be helpful if centres could provide candidates with practice of this. Nonetheless, many candidates achieved very high marks and the proportion of candidates who were unsuitable for this standard was relatively small. As with recent sessions, the use of additional sheets was uncommon and was usually limited to repeat attempts at parts of questions which were the most demanding. Similarly it was notable again that most of those that did need to repeat a solution indicated so clearly, which was very helpful to markers. A few, however, still leave a choice of answers which should be discouraged. Likewise centres should continue to encourage candidates that any "rough work" should be clearly indicated as such.

The majority of candidates presented very clear and accurate solutions throughout the paper, showing a good understanding of the mathematics needed for this module. Candidates were particularly successful with questions that were most similar to those from previous sessions, such as "disguised" quadratics, simultaneous equations and basic differentiation. A significant number of candidates were uncomfortable with the arithmetical demand of this non-calculator module with errors with fractions and negative numbers frequently seen. Similarly, the solution of quadratics where the coefficient of the square of the variable was greater than one proved demanding; reliance on the quadratic formula often led to large numbers that candidates found difficult to deal with. Accurate description/interpretation of the transformation of graphs remains a challenge, and the sketch required in question 8 seemed particularly unfamiliar.

Comments on Individual Questions

1)	Almost all candidates recognised the need to rationalise the denominator and many did so efficiently and accurately, securing all three marks. Arithmetical errors were sometimes seen both in evaluating the numerator and the denominator, more commonly in the latter e.g. by not squaring the 2 and ending up with a denominator of 5. A small but noticeable number obtained the correct answer but then proceeded to multiply by 3 to give integers for a and b; this error was not condoned. Likewise, a small number of those who chose to use $-\sqrt{7}-2$ as their multiplier, left their final answer with a negative denominator which was not deemed appropriate at AS level.
2)	Most candidates secured a large number of marks in this standard simultaneous equation question. Elimination of *x* tended to lead to more errors with the initial "*y* = " being lost and resulting in an incorrect quadratic. Whichever variable was eliminated, difficulties with fraction arithmetic regularly led to loss of accuracy marks, particularly with the negative value of *x*. Around two-thirds.of candidates scored full marks.
3)	This question tested both index notation and simple differentiation, with errors more common in the former process. Although almost all recognised that $\sqrt{x}=x^{\frac{1}{2}}$, a significant number incorrectly processed $x^2\times\sqrt{x}$ as x or $x^{\frac{3}{2}}$. The method mark was then still available for differentiating their expression, and was almost always earned. A small number of candidates used the product rule, which created a lot more work but was generally efficiently applied.

4)	Interpreting an equation given in "completed square" form proved quite taxing for many candidates with only 40% securing all five marks. Although most recognised the need to put $y = 0$ to obtain the roots, many multiplied out the given squared expression and then rearranged and factorised or used the quadratic formula. Those that then obtained a different incorrect quadratic received no further credit for trying to find the *x* intercepts. Candidates who used the given form were more likely to be successful. Many also correctly spotted the turning point from this form, although others resorted to differentiation. A significant number ignored the instruction to "give the coordinates" of the turning point and credit could not be given if this was not clear. Similarly, some omitted to state the *y*-intercept. An alternative approach of trying to construct the graph through a series of transformations from $y = x^2$ was seldom productive.	
5)	Although this disguised quadratic needed rearrangement as well as substitution, this question was well approached by the vast majority of candidates, with around 70% achieving all 5 marks. As in previous sessions, some candidates did not make their choice of substitution clear which made it difficult to award marks. The question was best approached by factorisation, and those who opted to use the quadratic formula were often unable to deal with the required arithmetic. Most remembered to cube their solutions to the quadratic, although some did so inaccurately, particularly the fractional solution.	
6)	(i)	Almost all candidates recognised the need to take out a factor of 3 in order to complete the square, and most also secured the second mark. The combining of constants remains a problem for many candidates, with fraction errors, sign errors and multiplication errors all regularly seen. Nonetheless, a large proportion – over 60% – secured all four marks.
	(ii)	Most candidates were successful in this part of the question, with only a few not recognising the word discriminant and a small number not stating the correct number of roots.
7)	(i)	Almost all candidates secured the first two marks in this part, but a significant proportion then failed to obtain all three roots of the cubic, either neglecting zero or $-\frac{1}{2}$.
	(ii)	Most candidates successfully used the second derivative to find the nature of their roots from part (i), gaining the method mark, but the accuracy mark was withheld if they did not complete the process for all three roots. Alternative successful methods included drawing a sketch of the quartic.
	(iii)	Even those successful in the previous parts seemed unsure of how to use their answers to determine where the function was decreasing. Many made no serious attempt, and those that scored often only had one region correct because of previous errors. Some gave single values of *x* other than regions, and others appeared to be identifying the region where the curve lay below the *x*-axis.
8)	(i)	Candidates seemed less familiar with the shape of $y = \sqrt{x}$ than with other curves tested in recent sessions. Indeed, many sketched reciprocal graphs. Those who did know the shape usually realised that $y = -2\sqrt{x}$ represented a reflection in the *x*-axis, although the quality of sketching was often lacking with many graphs clearly tending to a horizontal asymptote or showing the wrong curvature throughout.

	(ii)	This was the most successful part of the question with the majority of candidates securing both marks. Some earned one mark for translating to the right instead of the left, but those who translated vertically earned no credit.
	(iii)	The vast majority of candidates realised the transformation was a stretch, and used the correct word to describe it. Finding the scale factor proved much more challenging and many opted to try a series of stretches rather than the single transformation explicitly asked for in the question. Unusually, those who tried to find the scale factor by stretching parallel to the *x*-axis were often more likely to succeed.
9)	(i)	Most candidates used factorisation to start their solution to this quadratic inequality and chose the correct "outside" region securing all four marks. The notation used to describe the region was usually correct, although trying to describe two regions in a single inequality remains a fairly common error. Likewise, incorrect language such as joining the two sections with the word "and" still loses the accuracy mark. Sign errors in initial factorisation were not uncommon.
	(ii)	Close attention to detail was needed to ensure accuracy here, and many candidates produced clear full solutions. A large number of candidates differentiated the equation of the curve and equated this to the gradient of the line, although the use of 3 instead of -3 was a common error. Likewise there were sign slips in the subsequent attempts to find *x*, *y* and *c*. The other common approach was to equate the line and curve and use the fact that tangency implies one root and a zero discriminant. This was equally effective but similarly prone to sign error.
10)	(i)	There were many full accurate solutions to this question, although the testing of the identification of the centre of a circle but without that specific request proved taxing to some. There were both sign errors and division errors, with $(4, -2)$ being frequently seen. Slips also followed in finding the gradient of the line. Many candidates chose the origin as the point to use to find the equation, although the use of the centre of the circle or even A itself were not uncommon. Some candidates failed to give the final answer in the required form.
	(ii)	Candidates who made errors in the first part of the question were still able to score 4 out of 5 in this part as follow through and method marks were allowed from wrong centres. Just over a quarter of candidates were able to secure all 5 marks, but a significant minority were unable to access this part, either through not realising A was the opposite end of the diameter to O or not realising that the perpendicular gradient was needed to find the coordinates of B. Several found B through the use of a sketch rather than finding the equation of the perpendicular line. In general candidates who used a sketch were more likely to access, and succeed in, this question than those who did not.
11)	(i)	Most candidates realised the gradient of the given line was 6 to secure the first mark in this part. There were some errors in differentiating the equation of the curve and many who were successful in this equated their gradient with 6, rather than the negative reciprocal as required given that the line was parallel to the normal rather than the tangent. The arithmetical demand of the negative fraction and negative *x*-value prevented many high-attaining candidates from scoring full marks in this part of the question, with only around one third achieving all 6 marks. A number of candidates merely equated the line and curve, apparently not

		noticing that the given line was parallel.
	(ii)	The level of arithmetical demand again proved to be the greatest obstacle to complete success in this part, even if the correct value of k had been found in part (i). Another common mistake was to use the gradient of the tangent rather than the normal.

4722 Core Mathematics 2

General Comments:

The vast majority of candidates were well prepared for this examination, and able to make an attempt at every question within the allocated time. Most candidates were familiar with the topics being tested and could apply their knowledge to a variety of situations, including the unstructured questions. Candidates should be aware that they are expected to be proficient with C1 topics as well; some candidates were unable to apply their C1 knowledge to solve simultaneous equations or to manipulate indices and surds, including rationalising the denominator of a fraction.

Candidates should ensure that they show clear detail of the methods used so as to allow credit to be awarded. Whilst a correct solution from a correct equation will gain full credit, if solving an incorrect equation there must be detail shown if a method mark is to be awarded. There were several points on this paper where a quadratic equation had to be solved; if the quadratic is incorrect then simply writing down roots will get no credit. This is equally true when using limits in a definite integral; there has be clear evidence of the correct use of limits for the method mark to be awarded.

Candidates are expected to be proficient in the use of their calculators, including the use of brackets and fractions. They should also ensure that their calculators are in the correct mode, especially with questions involving trigonometry, but also whether the answer should be exact or a decimal approximation.

In questions where the answer is given, candidates should ensure that they show sufficient detail so as to be fully convincing in their method. They must also take care in the detail shown as any wrong working will be penalised, even if it results in the expected answer.

Comments on Individual Questions:

Question No.

1(i) This question proved to be a surprisingly challenging start to the paper. Most candidates were able to identify the need to use the cosine rule, and the majority of these gained one mark for stating a correct equation. Rearranging the equation was problematic for many, with a common error being for the $\cos 60^{\circ}$ to be moved across to the other side of the equation independently of the rest of the product. The other common error was for the $\cos 60^{\circ}$ to be applied only to the second term in the product, with $x(x + 2)\cos 60^{\circ}$ becoming $x^2 + x$. Sign errors were also common. The most successful solutions made effective use of brackets throughout the entire question. Candidates who started with the version of the cosine rule where $\cos A$ appears as the subject tended to be more successful in obtaining the correct quadratic. Candidates who obtained the quadratic equation were invariably able to solve it correctly, and also appreciate that the negative solution should be discarded.

(ii) The majority of the candidates were able to quote the correct formula for the area of a non-right angled triangle, and correctly use their value of x.

2(i) This question was very well answered, with many candidates gaining full marks. The most effective method was to write out the trapezium rule using exact values and then evaluate this on the calculator. Using decimal equivalents often resulted in a loss of accuracy in the final answer. The most common error was for candidates to use their calculator in degree mode rather than radian mode.

(ii) This part of the question was less well answered, due to a lack of precision in the explanations. Some candidates referred to the tops of the trapezia being under the curve, whereas others identified the areas that had not been included in the calculation. Either approach was condoned, as long as there was sufficient detail to be convincing. For the sketch graph, candidates were expected to provide a sketch of $y = \cos x$ with four trapezia shown. The most common error was to draw trapezia whose top vertices did not actually lie on the curve, and other errors included drawing just a single trapezium and even attempting to use $y = \sin x$ as the curve. Some precise and convincing solutions were seen, but these were in the minority.

3(i) This part of the question was very well answered, and the majority of candidates gained all of the marks available. Most candidates used the binomial expansion and made efficient use of brackets in obtaining a fully correct solution. The most common error was to either not use brackets at all, or to ignore the brackets that had been used earlier, resulting in an expansion where the powers of $^1/_2$ were incorrect.

(ii) Most candidates appreciated the link with the previous part of the question and could deduce the requested coefficient by considering the relevant two terms. Some candidates attempted a new expansion, but this tended to be less successful.

4 The majority of the candidates were able to provide a fully correct solution to this question. Most appreciated the need to expand the brackets before integrating, and were able to do so correctly. They could then use the given point to evaluate c and hence give a fully correct equation as their final answer. Some candidates were unable to apply the C1 index laws when expanding the brackets, but they could still gain credit later on in the question. There were very few instances of candidates attempting to use the equation of a straight line, which has been a common misconception in previous examination series.

5 This question was also very well answered, with many fully correct solutions being seen. Candidates coped well with the lack of structure in this question and were able to formulate an effective method to find the area of the segment. Most candidates could state two relevant equations, either by quoting the relevant formulae or by working in fractions of a circle. An uncertainty over terminology resulted in some candidates using 24 as the area of the triangle rather than the sector. Having obtained two correct equations, most candidates were able to solve them simultaneously although a significant minority were unable to simplify the indices involved. Candidates were familiar with the process for finding the area of a segment and many gained credit for doing so, even if there had been errors earlier on. A minority of candidates elected to work in degrees rather than radians throughout. A number manged to do so successfully, but the awkwardness of the numbers involved proved too much for others. Whilst there is nothing wrong with this approach, candidates should always attempt to use the most efficient method.

6 This fairly standard integration question was very well answered by many candidates. The integration was usually accurate, especially of the quadratic curve. Candidates were usually able to write the reciprocal curve in an appropriate form, but it was a relatively common error for the index to decrease rather than increase. Candidates were then able to use limits accurately to evaluate their definite integral. The most common approach was to find two separate areas, and then find the difference to get the shaded area. This method invariably resulted in the correct answer, whereas candidate who subtracted before integrating sometimes did so in the incorrect order. A minority of candidates decided to multiply through by x^3 before integrating, whilst the actual integration may be easier they did not appreciate that they were no longer dealing with the original functions.

7(a) Candidates continue to be proficient when using logarithms to solve equations, and this question was no exception with very few incorrect solutions being seen. Some candidates used logarithms with base 2 or 3, whereas others worked in base 10; all of these approaches were equally successful.

(b)(i) This part of the question proved to be more challenging, with candidates clearly being familiar with the rules of logarithms but not always able to apply the relevant rule correctly. Most candidates gained the first mark for rewriting $2\log_2 x$ as $\log_2 x^2$, but only the more able candidates could make further progress. The most common error was to remove the logarithms term by term, and others explicitly 'expanded' the logarithm before achieving the same result. The most common method was to combine the two log terms before removing the logs, but a number of candidates rewrote 1 as $\log_2 2$ to produce a single term on the left-hand side. Most candidates who correctly combined at least two terms then went on to produce a fully correct solution. A more creative approach was to use an index, base 2, as the inverse of one of the log terms and then use rules of indices to simplify to the required equation.

(b)(ii) Most candidates appreciated the need to substitute their equation from part (i) into the new equation, with some substituting into the given equation and others removing the logarithm before doing so. Whichever method was employed, only the most able candidates were able to correctly remove the logarithm. The most common error was for the right-hand side to remain as 0, and some candidates never even removed the logarithm before solving the quadratic.

8(a) Most candidates could set up two correct equations and solve them simultaneously to obtain the correct first term. The majority first found a value for the common difference, using this to then find the first term. Whilst most solutions were clearly detailed and easy to follow, some candidates struggled to express themselves mathematically and it was not always easy to follow their reasoning.

(b) This proved to be the most challenging question on the paper, and only the most able candidates were able to provide fully correct and detailed solutions. Most candidates were able to set up the correct initial equation of $ar^7 = 2ar^5$, but many struggled to find a numerical solution to this equation. Despite it being given in the formula book, some students could not quote the correct sum formula with an incorrect index of $n - 1$ being the common error. However, many students could indeed find the correct ratio, substitute into the sum formula and rearrange to find an expressions for a. The more astute candidates were able to make further progress, by simplifying $(\sqrt{2})^7$ and/or rationalising the denominator, but fully correct solutions were in the minority.

9(i) This part of the question was very well answered, with the majority of the candidates able to produce a correct product. Algebraic long division was the most common approach and candidates coped well with the lack of a quadratic term. Coefficient matching was less common, but was equally successful. Most candidates made explicit use of the factor theorem to show that $(2x - 1)$ was a factor, and this invariably had sufficient detail to be convincing. If using algebraic long division it was not sufficient to simply have a 0 on the bottom line; attention had to be drawn to the lack of remainder for the mark to be awarded.

(ii)(a) Candidates were clearly familiar with the relevant identities but many lacked the mathematical precision required for the marks to be awarded. The most common errors were to have the indices incorrectly placed and for the coefficient of 2 to disappear. Even if these errors were later corrected, the identity had to be fully correct at the point of use for the mark to be awarded. Candidates should also appreciate that each step in a proof should be clearly detailed; in some cases a number of steps were run together resulting in a lack of clarity of argument.

(b)(ii) Most candidates recognised the link with the previous part of the question and appreciated that they had to solve the cubic in $\sin 2\theta$. However a significant minority did not realise that this was related to the cubic that they had already factorised in part (i) and made a fresh attempt to solve, sometimes even attempting the use of the quadratic formula on the cubic. Many did recognise the link and attempted to use the root of ½, but this was sometimes equated to $\sin\theta$ rather than $\sin 2\theta$. Candidates who correctly stated that $\sin 2\theta = ½$ were usually able to solve the equation to find at least two roots, with only the most astute candidates giving all 4 roots as their answer. Candidates seem to be comfortable working with angles in radians in an exact form, and there were only a few instances of angles being given in degrees instead.

4723 Core Mathematics 3

General Comments

This paper proved to be a fair test of candidates' mathematical abilities. Questions such as the unstructured Q.5 and Q.8(ii) with its daunting-looking equation were generally answered well. More challenging requests occurred in Q.4, Q.7 and Q.9 but many candidates displayed admirable mathematical awareness to deal successfully with these challenges. It is pleasing to note that a significant number of candidates recorded full marks on the paper. There were plenty of familiar requests in the paper and it was only a tiny minority of candidates who struggled to record more than a few marks.

Lack of time did not seem to be a problem for candidates although there were questions, particularly towards the end of the paper, where unnecessarily lengthy solutions were sometimes produced. In many cases, a moment's thought to plan a solution would have been beneficial. Greater care with presentation would also have helped, both for the Assessment Specialists marking the paper and for the candidates themselves. It was not uncommon for candidates to misread their own work, perhaps because a minus sign was not clear or because a figure had been scribbled so casually as to be almost illegible.

Candidates must also be conscious of the fact that others are going to assess their work. Is it legible? Is it clear what is meant? An example occurred in Q.3(i) with the sketch. A candidate who first drew the straight line $y = 2x - 7a$ and then drew the requested $y = |2x - 7a|$ on the same diagram did not necessarily make it clear which was the answer. Marking is carried out online and the scanning process is sensitive to all that a candidate has written so candidates' attempts at emphasis or their use of different colours may not be evident to those marking the script.

Comments on Individual Questions

Q.1
This question was answered correctly by over half the candidates but there were a significant number of candidates who recorded no marks on this opening question. A few did not realise that the integrand involved $(2 + e^{\frac{1}{2}x})^2$ and some who did thought that the result of the integration involved $(2 + e^{\frac{1}{2}x})^3$. Attempts at expanding the expression sometimes went wrong with the square of $e^{\frac{1}{2}x}$ causing most trouble. A further common error occurred with attempts at integrating $4e^{\frac{1}{2}x}$. The final mark was occasionally not earned, either because of the absence of π from the final answer or because candidates resorted to a decimal approximation before an acceptable exact answer had been reached.

Q.2
Part (i) was generally answered very well with candidates using the formula for Simpson's rule accurately. Some stated the expression to be evaluated using logarithms and proceeded to produce the answer. Others set out the values to be used in decimal form, sometimes with the values presented in a table. Errors did occur, presumably the result of careless use of calculator. There were very few instances of candidates using the wrong number of strips or of associating the coefficients 2 and 4 with the incorrect *y* values. There were a few instances of candidates trying to 'simplify' the *y* values where $\ln 3 \ln 7$, for example, became $\ln 10$ or $\ln 21$.
Part (ii) was a slightly more challenging request and fewer than half the candidates were able to note that logarithm properties mean that the answer to part (ii) is –2 times the answer to part (i). Answers such as 708.6 and 0.0014 were occasionally seen. Some candidates used Simpson's rule again and those doing so had to produce the correct answer to record both marks. Some

candidates, having obtained the value –53.24 or –53.23 (either of which was acceptable) decided to drop the minus sign, often referring to an area below the *x*-axis. But this question involves a purely numerical request and losing the minus sign meant the loss of the accuracy mark.

Q.3
Part (i) was answered very well and most recorded two marks without difficulty. A few graphs did not extend into the second quadrant and there were some errors with the two intercepts, usually involving the absence of *a*.

More than half the candidates recorded three marks for part (ii). Candidates considering two linear equations or inequalities tended to be more successful than those whose approach involved squaring. The latter sometimes omitted to square $4a$ and there were problems in dealing with the quadratic expression involving both *a* and *x*. Many candidates earned the first two marks but not the third one; the values $\frac{3}{2}a$ and $\frac{11}{2}a$ were found but the final conclusion of $\frac{3}{2}a < x < \frac{11}{2}a$ eluded them. A look back at their graph in part (i) would have helped but not many candidates associated their work in part (ii) with the earlier graph.

Approximately half of the candidates answered part (iii) accurately, concluding with the single integer 3827. Answers such as $N \le 3827$ did not earn the accuracy mark because this does not answer the question as asked. A number of candidates seemed unsure of the definition of 'integer' and offered an answer such as 3827.6; others were content to claim that the largest integer is $e^{8.25}$. It had been anticipated that candidates would answer this part by relating $\ln N$ to the upper limit of their answer from part (ii) with *a* replaced by 1.5. But this approach was not noted so often and the more common approach was to start again. Many candidates seemed curiously unaware where this largest integer would originate and explored the whole double inequality involving $\ln N$ before deciding on their answer.

Q.4
The vast majority of candidates had no difficulty in using the appropriate identity and solving the equation to find the two possible values of $\tan\theta$. Candidates correctly reaching the values –4 and $\frac{2}{3}$ earned all three marks at this stage; the penalty for proceeding with the incorrect value would follow in part (ii). In fact many candidates were unable immediately to choose the correct value and had to go further to find angles before making a choice. Others explicitly rejected –4, stating that the value is not between –1 and +1 or using their calculator to find the angle $-76°$ and observing that this is not in the required range.

For part (ii)(a), the vast majority of candidates knew the correct identity to use but only about half substituted the correct value of –4. Candidates offering two answers, using the values –4 and $\frac{2}{3}$, earned only the method mark.

Candidates did not fare so well with part (ii)(b) and statements such as $\cot(2\theta+135) = \frac{1}{\tan}(2\theta+135)$ and $\cot(2\theta+135) = \cot 2\theta + \cot 135$ were occasionally seen. Rather than using their value of $\tan 2\theta$ from part (ii)(a), some candidates endeavoured to set up an identity for $\cot(2\theta+135°)$ in terms of $\tan\theta$. Candidates were required to supply sufficient detail in their solutions to indicate that calculators had not been used and most did indeed do so. Just over a third of the candidates succeeded in reaching the correct value of $-\frac{23}{7}$.

Q.5
Many solutions to this question were impressive with candidates negotiating their way through the various steps with assurance. Approximately half of the candidates recorded full marks on this question. Most candidates handled the differentiation and integration accurately and

appreciated the need to find the point of intersection of the normal with the *x*-axis. There was some uncertainty over the limits to be used for the integration and a few candidates concluded by subtracting the area of the triangle from $\frac{125}{6}$ rather than adding it to $\frac{125}{6}$. It was surprising how many candidates calculated the area of the triangle by integration rather than carrying out a simple $\frac{1}{2}\times 2\times 5$ calculation.

Q.6

Parts (i), (iii) and (iv) were answered well with, in each case, approximately 80% of the candidates recording full marks. In part (i) the acceptable terms 'translate' and 'stretch' were almost always used and the accompanying detail was usually correct, if not always expressed perfectly. There were no problems with part (iii) and the vast majority knew that reflection in the line $y = x$ is the geometrical link between the two graphs. In part (iv), a number of candidates calculated $\mathrm{ff}(2)$; whether this was their genuine attempt at the request or whether it was the result of carelessness was not clear. Most candidates formed an expression for $\mathrm{ff}(x)$ and, with due care in most cases, proceeded to solve the appropriate equation accurately. A pleasing, though rather small, number of candidates demonstrated awareness of function theory by using the inverse function from part (iii) to find the value of *x* in part (iv).

Part (ii) was not done at all well and only about one-fifth of candidates earned both marks. Some earned one mark by providing half the answer but many seemed to proceed without giving any heed to the given domain of f. Some sketches, evidently of $y = \frac{1}{x}$, were in evidence often yielding a conclusion such as $y < 0,\ y > 0$. Another answer often seen was $y \geq 3$, presumably based on an assumption that *y* increases from its value when $x = 0$ as *x* increases. A little thought to produce a sketch of the graph for the appropriate domain should have led to greater success. Alternatively, informed use of a graphical calculator would have helped.

Q7

Part (i) was not answered well and only about one-quarter of the candidates recorded all three marks. The question is specific in the approach to be taken and other methods, such as those involving implicit differentiation, did not earn any credit. For many candidates, the first mark for expressing *x* in terms of *y* was the only mark earned. The problem then was that many failed to appreciate the implication of *a* being a constant. Attempts using the quotient rule for differentiating $\frac{\ln y}{\ln a}$ in which $\frac{1}{a}$ appeared as the derivative of $\ln a$ were very common.

Attempts at part (ii)(a) were also disappointing and only about one-third of the candidates earned all three marks. Many candidates failed to see any connection with part (i) and, for them, the derivative of 4^x was often $x \times 4^{x-1}$. Others, with probably more than half an eye on the given answer, decided that the derivative was $4^{x-1}\ln 4$.

A request involving iteration is usually answered very well in this unit but this was not the case this time. The sequence converges quickly and candidates were expected to show the iterates to at least three decimal places to justify their answer of –1.27 for the *x*-coordinate. Many then seemed to forget the need to find the *y*-coordinate. Many others used the value of –1.27 to calculate the value of the *y*-coordinate, and this leads to the inaccurate value of 2.77. Some candidates were guilty of careless calculator use in claiming a *y*-coordinate value of –2.43; they did not seem to be concerned that this did not accord with the position of *P* indicated by the diagram.

Q8

Part (i) was generally answered very well with approximately 80% of the candidates recording five marks. Candidates generally had no difficulty in using the appropriate identity with the correct definitions of $\sec\theta$ and $\operatorname{cosec}\theta$ to reach the expression $6\sin\theta + 8\cos\theta$. Reaching the required form from this was a familiar process. There was occasional uncertainty in finding the angle α and candidates who did so via the erroneous statements $\cos\alpha = 6,\ \sin\alpha = 8$ did not earn full credit.

Candidates had more difficulty with part (ii) but it was encouraging that approximately 40% of candidates recorded full marks on this part. Most candidates took account of the word 'Hence …' and realised that using $\theta = \beta + 10°$ was the appropriate link with part (i). A minority of candidates saw no link with part (i) and made no effective progress whilst a few others proceeded with an incorrect equation such as $10\sin(2\beta + 73.1°) = 3$. For those candidates embarking on the solution of the correct equation $10\sin(\beta + 63.13°) = 3$, there were two values to be found and a significant number of candidates managed to find only one. There were also candidates who managed to find three values in the required range; uncertainty over how to deal with the initial value of $\beta = -45.67°$ was the problem.

Q9

Success in both parts of this question needed both an awareness of the various mathematical techniques to be used and careful organisation of the expressions involved. In many cases, a more thoughtful and considered approach would have made the work much easier.

Many candidates earned the first two marks of part (a) for correct use of the quotient rule. The appropriate quadratic equation was then produced although often there was a sign error on the way. Most candidates then struggled to produce a convincing conclusion. Some merely stated that a quadratic equation has two roots. Others substituted into the quadratic formula and felt that was sufficient. Many did concentrate on the discriminant but, in many cases, just stated that it is positive and so there must be two stationary points. A rather more persuasive argument was required.

A significant number of candidates made no progress with part (b) and some were unable to devise a strategy. Many others did earn the first two marks with accurate work in finding the first derivative. Indeed it was pleasing that many were able to differentiate the awkward e^{x^2} correctly. Further success usually depended on candidates organising their first derivative into a form suitable for further differentiation. Candidates who tried to differentiate a term such as $2xe^{x^2}(ax^2 + b)$ occasionally succeeded but generally were unable to cope, either failing to use the product rule appropriately or making careless slips. Candidates who organised their first derivative into a form such as $e^{x^2}(2ax^3 + 2ax + 2bx)$ were then faced with a manageable task to find the second derivative. Some of the candidates who reached an expression for the second derivative did not realise the implication provided by the form of the second derivative given in the question and were unable to conclude successfully. But it is pleasing to record the fact that approximately one-fifth of the candidates did manage to prove the result.

4724 Core Mathematics 4

General Comments:

The paper proved accessible to almost all of the candidates, but there was also enough to stretch the best and full marks was rarely achieved. There were many examples of well-presented responses and some excellent work was seen. However, there were also some instances of poorly presented work and handwriting that was very difficult to read – in some of the more extreme cases it seemed that candidates couldn't read their own writing as there were many examples of miscopied work.

Many candidates demonstrated a good understanding of Core 4 syllabus material, but failed to do themselves full justice in the examination either because they didn't answer the question fully or because of poor algebra – bracket and sign errors were common - and careless arithmetical slips.

A surprising number of candidates were unable to relate work done in the early part of a question to the demand of a subsequent part. It is advisable to read through the whole question first rather than attempt a solution in a piecemeal fashion.

Comments on Individual Questions:

Question No. 1

Part (i)

This proved accessible to nearly all candidates, with most scoring full marks. A few made arithmetical slips, and a few used the wrong index: – 4 and ½ were the usual errors.

Part (ii)

Although largely well done, this did cause some problems. Common mistakes were to omit one tail of the inequality or to write $x < \left|\frac{1}{8}\right|$. Occasionally $|x| < 8$ was seen.

Question No. 2

Most candidates scored full marks on this question. Very few were unable to make some progress, and those that went wrong usually did so through careless slips.

Question No. 3

This was question was done very well, with many candidates achieving full marks. A few candidates integrated *x* when applying integration by parts, and more often than not the correct result mysteriously appeared from wrong working.

Question No. 4

Nearly all the candidates were familiar with this topic, and most went on to score full marks. A few slipped up with the arithmetic and obtained one or sometimes two incorrect coefficients. Those who attempted a solution by equating coefficients went astray more frequently.

Question No. 5

Part (i)

This was done very well indeed. Most candidates opted for long division, but other methods were often just as successful.

Part (ii)

Most candidates realised that they should use the result from part (i) but many then stumbled with integrating the rational part of the integrand.

Question No. 6

As in previous years, this topic is well understood and there were many very good responses to this question. A few slipped up in finding the values of *x*, but most differentiated successfully and then rearranged to make $\frac{dy}{dx}$ the subject of their equation. A significant minority made a sign error at this point, thus losing the accuracy,marks at the end following substitution of usually correct *x* values.

Question No. 7
A minority of candidates floundered from the start, either combining the fractions and then integrating incorrectly, or simply failing to spot the logarithmic form. A few candidates also integrated the right hand side incorrectly, $\frac{k^2}{2}$ being the usual error.

Most earned the first method mark, but some made a sign error on the left hand side, and some missed out the constant of integration. Both errors were costly and inhibited further progress. The majority went on to find the constant of integration successfully, and made progress with combining the logarithms correctly. Exponentiating both sides was often less successful, however. Very few candidates were able to find a correct final expression for *A*.

Question 8
Part (i)
A variety of approaches were seen. Most went straight to the quotient rule and the chain rule and went on to derive the given result successfully. Almost as many separated the logarithms and used the chain rule before combining to a single fraction, and they were equally successful. A few converted to the reciprocal forms and were generally successful. A significant minority made slips with coefficients, signs and brackets, thus losing the accuracy marks. Candidates are reminded of the need for rigour when deriving a given result.
Part (ii)
Candidates who failed to combine the integrand into a single fraction generally made no progress.
Of the good number who did adopt the correct strategy, a significant proportion made sign or coefficient errors and so were unable to make the connection with part (i).

Question 9
It was pleasing to see so many near perfect solutions to this tricky substitution question. The majority of candidates were able to make a start, although the amount of progress, but many candidates were not able to make much progress. Most correctly differentiated *u*, but often failed to rearrange the expression correctly. Substitution in the numerator for $1 - \ln x - x$ defeated many and bracket and sign errors were commonplace.

Question 10
Part (i)
Most knew what to do here. A few candidates wrote an expression and not an equation, or missed out the parameter, thus losing an easy mark.
Part (ii)
This was done well by most candidates. A few made errors in calculating the lengths, and some calculated either the external angle of the triangle or angle *APB* or *ABP.*
Part (iii)
Many candidates did not know where to begin and often failed to score. However, a variety of successful approaches were taken and some excellent work demonstrating thorough understanding was seen.

Question 11
Part (i)
This was done well by most. A few slipped up with $\frac{dx}{dt}$ or made bracket or sign errors when combining the two derivatives.
Part (ii)
Most were able to start this part, but often went stray in finding the correct value of *t*.

A significant minority of candidates worked with $t = -2$ and / or $t = 1$, which was pointless as both values are outside the specified range. Even those who did work with $t = -1$ only often went astray in finding x and y. Only a few candidates realised the need to check the x-values as well as the values of the gradient when determining the nature of the stationary point, and those who tried to use the second derivative almost invariably went wrong.

Part (iii)

Very few candidates were able to state both ranges correctly.

Part (iv)

Only a very small minority sketched the curve successfully. A significant proportion of those who were successful had not managed full marks in the previous parts of the question.

4725 Further Pure Mathematics 1

General Comments:

Most candidates were able to attempt the majority of questions, showing a sound range of knowledge of the specification content. Completely correct solutions to all questions were seen, and no question proved inaccessible to the candidates. There were very few times when a candidate made no attempt at a particular part of a question. The standard of presentation was generally quite good, but poor handwriting often lead to marks being lost through simple arithmetic or algebraic errors when the correct method was known.

There was no evidence of candidates being under time pressure. However, as has been mentioned in previous reports, in a number of questions there was the opportunity to check a solution, but few candidates did this and so could not identify that an error had occurred and have the chance to rectify it.

Comments on Individual Questions:

Question No.

1 Most candidates used the correct standard results, the most common error was to give the last term as 8, rather than 8*n*. This inevitably meant that their result could not be factorised. A significant number of those who found a correct expression, did not see the factor of 2, or did not factorise $(n^2 - 25)$.

2 The method for finding the square roots was generally well known, but often a sign error at an early stage of the working meant that the final answers were incorrect. This is a situation where the error could have been spotted by checking answers squared back to the original given complex number.

3(i) This part was answered correctly by the majority of candidates.

3(ii) By simplifying the given expression to **BA**, the correct answer was usually obtained. Some gave the expression as **AB**. Those who attempted to find $\mathbf{A}^{-1}$ and $\mathbf{B}^{-1}$ and multiply the results invariably failed to deal with the determinants correctly.

4 Some candidates failed to show clearly that the result is true for $n = 1$. Most added the correct next term to the given sum, although a significant minority added $\frac{1}{3}$, the value of the first term to the given sum. A significant number of candidates did not give a clear explanation of the induction process, so the last mark was often lost.

5(i) The most common error was to give transformation P as a stretch. Those who recognised a shear, often gave a "scale factor" rather than the image of a particular point. The word "factor" implies that all values (of *x*) are multiplied, which is not the case in a shear.

5(ii) This part was generally answered correctly, with a stretch parallel to the *x*-axis being a common error.

5(iii) The matrix multiplication was shown correctly by most candidates, the most common error was finding **PQ** rather than **QP**.

5(iv) Most found the determinant of their answer to (iii), while some used the scale factor for each transformation and others gave a convincing diagram to obtain the correct answer.

6(i) In the sketch, many candidates did not make a sufficiently good attempt at the positioning of z_1 and z_2, not being able to deal with the angles, which are given in radians, so that z_1 was in the second quadrant, and towards the imaginary axis, while z_2 is in the third quadrant and towards the negative real axis.

6(ii) Poor attempts at (i) meant that many did not see that OZ_1Z_2, for example, was a right-angled triangle, which makes the calculation of the modulus and argument of $z_1 - z_2$ reasonably easy. A significant number converted all 3 complex numbers to a decimal form, which will not give the exact values of the modulus and argument as required.

6(iii) Most candidates recognised that the locus was a straight line, but many failed to give sufficient detail to indicate the correct perpendicular bisector. .

7(i) The majority of candidates showed sufficient working to justify the derivation of the given answer. A numerator of $2r + 5 - 2r - 1$ was the most common error.

7(ii) Those who started the difference method at $r = 2$ generally found the correct cancelling and then the correct value to 3 decimal places. Those who started at $r = 1$, usually made an error in the cancelling or did not deal with the subtraction of the first term from their expression properly.

7(iii) A sensible attempt at (ii) usually meant a correct answer was given for this part.

8 Many different approached were seen, usually with some or complete success. The most frequent error was the omission of the negative sign in the appropriate symmetric functions, or in failing to deal with the co-efficient of x^3, or sometimes both.

9(i) The method for finding the determinant of a 3×3 matrix was demonstrated by virtually all candidates and most could solve the associated quadratic equation successfully.

9(ii) The explanation of the unique solution was often too vague to be awarded the mark.

9(iii) Most candidates attempted to solve the equations by an elimination method, but some eliminated e.g. z, but then eliminated y, so were unable to set up an equation from which p could be found. A reasonable number saw the linear connection between the three equations and so found the value of p easily.

10(i) The imaginary parts of both expansions was generally found accurately and sufficient working was usually shown to justify the given answer.

10(ii) In the expansion of z^*w, the term $-\mathrm{i}^2bd$ frequently became $-bd$, which meant that accuracy marks were lost. Some candidates eliminated b from their equations, rather than d, any many who eliminated correctly found only one value for b in terms of a, the ± being omitted.

4726 Further Pure Mathematics 2

General

There was a slight increase in the number of candidates this year and the mean mark was a little higher. Most candidates seemed to be able to tackle all the questions.
As with last year, the sketching of graphs left a lot to be desired. Asymptotes parallel to the *y*-axis, for instance, in question 4(iv) were often not straight. Additionally, the sketching of the curve was supposed to show that the curve approached the asymptote, but often it curved away. A diagram for question 9(i) would have helped enormously in the description of what has to be demonstrated, but was often so poor as to be of little use. In question 5(i) careful thought of what the shape of the curve might be, including the sections where *r i*s positive, would have helped.

Question 1
This first question was mostly done well. A few candidates thought that $\cosh x$ differentiated to give $-\sinh x$ and some used exponentials but then could not simplify $e^x = 5e^{-x}$. It was acceptable for the formula for $\tanh x$ from the formula book to be used and so the conversion to exponentials was a rather long winded method.

Question 2
Most completed the square correctly but some could not then use this to carry out the integration. Several sign errors, some who could not decide between $\tan(x-3)$ and $\tan^{-1}(x-3)$ and some who thought that the answer needed to involve $\ln(x^2-6x+10)$. A small minority who managed the integration gave an answer in degrees rather than radians.

Question 3
Some candidates found part (i) very difficult and several used additional sheets, or even additional answer books, for this part. The candidates who realised that they had to use the chain rule were usually fine, apart from occasional sign errors, but many candidates ignored the change of variable and just wrote the answer as $\dfrac{1}{1+\left(\dfrac{x}{1+x}\right)^2}$, often spending time trying to simplify this and then using the quotient rule to try to find the second derivative. For part (ii) some candidates tried replacing *x* by $\left(\dfrac{x}{1+x}\right)$ in the Maclaurin expansion of $\tan^{-1}x$, but most realised that having found the first and second derivatives they knew enough information to write down the expansion as far as the term in x^2.

An alternative approach was to rewrite the equation as $\tan y = \dfrac{x}{x+1}$ and to differentiate implicitly. Some immediately found $\dfrac{dy}{dx}$ when $x = 0$, then moved on to find $\dfrac{d^2y}{dx^2}$ implicitly and hence the value of $\dfrac{d^2y}{dx^2}$ when $x = 0$.

Others substituted for $\sec^2 y$ back into *x* first and had to do some heavy algebraic manipulation to achieve the given answer; inevitably many made errors in this working.

Question 4

The vertical asymptotes were usually correct. Some candidates did not realise that there was also a horizontal asymptote and some stated that $y = x$ was an oblique asymptote. Most candidates identified the point (1, 0), even when they had not given $y = 0$ as an asymptote.
A common approach in part (i) was to split the function into partial fractions. This does not reveal any extra information which could not be found from the original form. Furthermore, in part (iii) these candidates rarely followed through the differential in this form to argue why the gradient function was never zero, but put it all back together again to obtain a single fraction and an argument that the numerator of this fraction could never be zero.
For those who did not follow this path then the approach for part (iii) was to differentiate the quotient and to argue that if there was a turning point then the numerator of this fraction would be zero. Some candidates did not then explain why this could not be so or what the consequence of this was for the original curve.
When sketching graphs of this nature, it should be expected that candidates will draw the asymptotes, give the coordinates of any turning points and the intercepts on the axes. Given that the lines $x = -1$ and $x = 2$ were going to be drawn, most candidates drew axes with a scale on the x axis. It could then be seen that the curve crossed the x -axis at (1, 0). It was, however, also expected that the intercept on the y-axis would be given and if there was no scale on this axes (and there usually was not) then the coordinates of the intercept should have been given.
In spite of earlier work done, a number of candidates failed to show all three sections of the curve and so earned no marks in part (iv).

Question 5

The sketches were often not neatly produced and the intercepts with the boundaries where $\theta = 0$ and $\theta = \frac{1}{2}\pi$ were rarely marked. The polar coordinates of the intersection were often correct; candidates who had drawn the part of $r = \cos 2\theta$ with negative *r* sometimes also gave $\left(1, \frac{1}{2}\pi\right)$. Sections of the graphs for which $r < 0$ were not penalised at this stage, but inevitably caused confusion in later parts. The determination of the area was sometimes correct, but many candidates integrated the part involving $r = \cos 2\theta$ with an upper limit of $\frac{1}{2}\pi$ instead of $\frac{1}{4}\pi$.
Diagrams to show what area was being calculated helped candidates to keep track of their working.
Candidates producing two areas based on the same limits had clearly not understood the underlying techniques and it was not unreasonable that this was reflected in the marks awarded for this part of the question.

Question 6

Most candidates knew what was required of them in part (i) though many did not set out their work very well. Some candidates wrote expressions involving exponentials but did not connect them to the hyperbolic functions. Many candidates were unable to expand $\left(\mathrm{e}^{x} - \mathrm{e}^{-x}\right)^{3}$ directly, instead squaring first and then multiplying out with the third factor. This extra unnecessary algebraic manipulation usually gave rise to more errors.
In part (ii) full marks were given to the expression in the correct format for either x or w and very few failed to gain full marks for this part.

Question 7

In part (i), candidates realised that they needed to use integration by parts. Those who chose initially to define “u” and “dv” the wrong way round soon got themselves into trouble and they were then unable to realise their error, choosing instead to give up the question.
Part(ii) was found difficult by many candidates who did not demonstrate what was required conclusively.

In part (iii), most candidates found I_0 but a few made more work for themselves by finding I_1. As with part (i) there were often some extra negative signs around which disappeared without mention. These solutions did not gain full marks since the argument was not complete.

Question 8

Part (i) should have been done using change of sign, and often was. Some candidates did not give the numerical values of the expressions and just said 'positive' or 'negative' and some worked out the values but did not show how this led to the conclusion. In part (ii) many candidates failed to read the question carefully. Having found the first three iterations as required (though not always to 5 decimal places) they then continued to carry on iterating in order to find a value for β and so did not answer the question relating to justification of the value from the first three iterates.

In part (iv), determining the convergence or divergence of the iterative formulae was best done using gradients or a web diagram. Those that chose to demonstrate a failure to achieve the required root by carrying out iterations often gave insubstantial working to demonstrate conclusively what was required.

Question 9

A diagram showing the rectangles was the best way to start this question. Without a diagram it was difficult to explain the result in a convincing way that did not look like just writing out the expanded version of the given result. Far too many candidates assumed that *n* was 10. Parts (ii) and (iii) were done fairly well although some candidates tried to start their summation for the lower bound from $r = \frac{1}{n}$ instead of $r = 1$. Those who were able to write down the lower bound summation were usually able to subtract the two to get a first and last term, though the solution of the resulting inequality caused difficulties for some.

4727 Further Pure Mathematics 3

General Comments:

Candidates were generally well-versed in the standard methods necessary for this paper. They also competently handled the easier elements of problem-solving. The biggest challenge faced was in clear mathematical communication. This was reflected in weaker answers to questions of demonstration, particularly in the case of group theory. The issue with this latter topic is that it requires a level of precision and of coherent logical argument which was beyond many candidates.
Some candidates, who otherwise performed well, showed weak levels of algebraic manipulation when dealing with rational terms.

Comments on Individual Questions

1 Most candidates had been well prepared for solving differential equations. They recognised the need for an integrating factor and could apply this method. Most, also, ensured that their constant of integration was included in the process of rearrangement to reach the form "y = …".The most common error was miscalculation of the constant due to poor algebraic manipulation.

2(i) Candidates handled this group table using modulo arithmetic with ease.

(ii) In determining whether the groups were isomorphic, the majority of candidates determined the orders (1,2,2,2) of the elements of G and of *H* and then asserted that the two groups are isomorphic since the orders match up. This is an incorrect principle, which fails for groups of higher order, and so they could not gain full credit, unless they went on to point out that, up to isomorphism, there is only one non-cyclic group of order 4. Alternatively, they could state that both groups were copies of the Klein group. An alternate answer which avoided this pitfall was to draw up the full group table of *H* and show an explicit isomorphism between *G* and *H*.

3 Many candidates were able to reach the correct solution with ease. The commonest errors included:

- $\lambda = 3$ leading to $y_c = (A + Bx)e^{3x}$
- $y_c = Ae^{-3x}$ or $(A + B)e^{-3x}$
- miscalculation of a, b in the particular integral
- omission of "$y =$ …" in the final answer.

4(i) Almost all candidates appeared familiar with one of the methods for deriving the equation of a plane through 3 (non-collinear) points. However, a number of candidates lost marks through failing to give sufficient justification. It is important at this level, when asked to "show that" (rather than simply to "find"), to communicate your method mathematically. Candidates were expected to explicitly show the required vector subtractions for obtaining the direction of two vectors in the plane, and to exhibit at least something of the geometrical context, such as by reference to the "normal" vector.

(ii) Many candidates proficiently found the line of intersection; the usual method being to calculate the cross product and find a particular point. Some did it by eliminating a variable from the pair of plane equations and then writing x, y, z in terms of a parameter, but this approach was more prone to errors of calculation or errors when converting their parametric form into vector equation form.

(iii) The angle required was usually correctly found. The only notable error was to give the complementary angle, either by misuse of sine in the scalar product or by confusing the method required here with the one for finding the angle between a line and a plane. A few candidates when finding angles in this paper insisted on using the lengthier method of using the vector product rather than the scalar product and, although this is acceptable, centres may wish to stress the time-saving benefits of the latter.

5(i) This question was generally well answered. Most candidates attempted to use the $\frac{1}{2}ab\sin C$ formula with the correct angle. Some successfully used $\frac{1}{2}\text{base}\times\text{height}$ (although they tended to sketch an argand diagram with real *z* which simplified their task). Common errors were to have sides of 5 and 5, rather than 5 and 10, or to use the complex numbers, rather than their moduli, for the sides.

(ii) Some good Argand sketches were seen, with the best ones having realistic gradients for each PR and showing the arc, centred P, through Q. However, very many were untidy, with R_1 and R_2 poorly positioned. The lengths of PR_1 and PR_2 were often very different from that of PQ and the angles drawn were frequently far less or far more than $\pi/4$. The second part was badly done by many candidates, with some failed attempts at using trigonometry. The best answers converted rotation and translation into complex algebra. Some candidates scored partial credit for finding $(4+2i)e^{\pm i\pi/4}$, but failed to complete the argument by adding $-1+i$. One alternative approach which candidates often couldn't complete involved writing $\overrightarrow{PQ}$ in the form $Re^{i\theta}$ before adding $\pm\frac{\pi}{4}$ to the argument. Some candidates simply, erroneously, multiplied the modulus of $\overrightarrow{PQ}$ by $e^{i\frac{\pi}{4}}$.

6(i) Many candidates remain unaware of the existence of the formula for perpendicular distance from a point to a plane which is given in the List of Formulae(MF1). Those who used the standard formula were mostly successful although some got the sign of "d" wrong. Using other (lengthier) methods such as finding the parameter for the base of the perpendicular worked well for some candidates, but others made numerical errors or didn't have a complete method to hand.

(ii) The standard method for finding this angle was familiar to many candidates. Even those who were unsuccessful could usually gain the first mark by finding the angle between line and normal to plane, even if they did then take this to be the solution.

(iii) This question was one of the best answered ones in the paper. Almost all candidates successfully deduced the equation of the line, found the value of the parameter and used it to find the co-ordinates of the point. Most dropped marks resulted from calculation or transcription errors.

7(i) A good number of candidates scored full marks on this part. Marks were lost by those who omitted to write equalities between successive lines, or did not show the grouping of exponentials into pairs, or who worked exclusively with *z* without ever defining it as $e^{i\theta}$. The best answers started with the standard formula $cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$ and then defined *z* to aid conciseness. Some candidates still insist on expanding binomials longhand when it is expected that they should use the formula.

(ii) The majority of candidates successfully employed the identity from 7(i), together with the double-angle formula, to convert the equation to $32\cos^6\theta = 26\cos^2\theta$. From that stage onwards there was a divide between strong answers that tackled general equation-solving well and those that made standard errors. One error was to lose a solution by dividing through by $cos^2\theta$ rather than taking it out as a factor. The other common error was failing to find the two solutions in the range to $\cos^4\theta = \frac{13}{16}$ due to neglecting the negative fourth root.

8(i) This was another question which was mostly answered correctly. Occasional wrong answers of 4, 24 or 256 were seen.

(ii) Most candidates addressed the conditions necessary in a subgroup, but answers were almost always insufficiently clear or precise. In checking for identity, closure and inverses, the commonest deficiency was to fail to say that elements belonged to *H*, sometimes, instead, saying that they belonged to *G* or "the group". The best answers addressed:

- closure by considering the product of two general elements of *H* and concluded that the product was still in H,
- existence of identity, concluding that it was also in H,
- existence of all inverses by computing the inverse of a general element and concluding that it, too, was in H.

It should be noted that consideration of associativity is irrelevant for subgroups, this being automatically inherited. It did not, however, draw any penalty, except in the case where the candidate was clearly considering commutativity rather than associativity. A more important consideration (unnecessary here) is whether the set in question is a subset of the parent group. It should also be noted, in consideration of closure, it is inadequate to simply consider the square of a general element.

(iii) These last two parts challenged even the strongest candidates. Many candidates correctly attempted part (iii) by considering the possible orders of subgroups. However, there was a lack of precision of mathematical argument in many cases. It was crucial to explain explicitly that the order of *K* is a factor or divisor of $|G|$ (=16), and that the order of K is a multiple of $|H|$ (=4). To completely justify the solution of "8" it was also necessary to explicitly link the "properness" to the elimination of the other possibilities rather than conflating it with Lagrange. There were many attempts without the above elements merely producing lists of possible orders for *K* and then whittling them down to 8.

(iv) This demonstration was treated adequately by only a tiny number of candidates. Many candidates gained one mark for identifying the elements of *K*, and a possible second mark for making a start on trying to show that *K* could not contain any matrix of the form $\begin{pmatrix} \pm i & 0 \\ 0 & b \end{pmatrix}$. The attempted arguments here were invariably logically unsound, and could therefore not gain credit in a topic of this sort. The point almost universally missed was that if *K* contained a particular matrix $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, then it follows by multiplying by a general element of *H* that *K* contains $\begin{pmatrix} a & 0 \\ 0 & c \end{pmatrix}$ for *all* *c*. Consequently, to show that if *K* contained some $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ with $a = \pm i$, it would also contain the square $\begin{pmatrix} -1 & 0 \\ 0 & b^2 \end{pmatrix}$, was to show that it would generate an extra 8 elements in addition to those of *H* , thereby making the order of *K* too large.

4728 Mechanics 1

General Comments:

Many candidates coped well with the demands of this mechanics paper. Though the difficulties encountered could be related to the syllabus, often it was the back-up skills from pure mathematics or calculator use which impeded candidates.

Questions 3(i) and 7(iii) did cause problems through the use of simultaneous equations. Questions 5(i) and 5(ii) needed accurate calculator use. Question 7(i) incorporated a situation exploiting sin = opp/hyp, which even some better candidates did not perceive.

Another area of concern was the comprehension of mathematical language. Q6(ii) found some candidates using 0.4 as a coefficient of friction, not a frictional force. Q6(ii) and Q7(i) exposed confusion between the contact force and its normal component. The frequent use of g in Q1 may have been a consequence of confusing "*A* above the level of *B*" with "*A* above *B*". Had some candidates read and re-read questions with sufficient care, their performance could have been better. In Q1, Q7(i) and Q7(ii) this might also have led to all quantities requested being found.

Comments on Individual Questions:

Question No. 1

Candidates sometimes forgot that, on a smooth inclined plane, the acceleration of a particle is less than g. In such cases both M1 marks were awarded to answers "correctly" using an acceleration of 9.8 m s^{-2}. There were also candidates calculated only one of the two values requested.

Question No. 2

Compared with previous sessions a significantly higher proportion of candidates using the cosine rule were able to gain full marks for the resultant. However, having used the sine rule to find a correct angle these candidates were prone to lack the understanding of how to relate their angle to the required bearing. Candidates who worked entirely from components were more likely to find the bearing of the resultant correctly.

Question No. 3

In part (i) a high proportion of candidates gained the first three marks for setting up correctly a conservation of momentum equation. Correctly using the distance and time information was not common. Weak candidates thought the question related to *suvat* formulae. The most successful approach was using v and (v+0.8) as the "after" velocities in the original momentum equation. Candidates doing this had a fully thought out strategy for answering the question. Much more complex was setting up and solving two simultaneous equations, typically $2.46 = 0.2u + 0.3v$ and $2v - 2u = 1.6$, where sign errors were frequently seen. Perhaps inevitably some solutions were based on the idea that the particles moved in opposite directions, and the (strictly incorrect) answer of -4.44 m s^{-1} for the final velocity of *A* was given full credit.

Part (ii) was sometimes marred by working with the total momentum of both particles. Those candidates who found the final velocity of *A* to be negative struggled to decide how many minus signs to include in their equation for momentum change.

Question No. 4

In part (i), candidates were asked to "verify" t =1.5 which indicated that finding the distances moved by the object and ball should first be calculated. The next stage was to demonstrate that these distances added together equalled the 27 m initial gap. This stage had to be done using exact arithmetic; the use of rounded figures could lead only to the conclusion that the ball and object were very close. Some candidates successfully set up and solved and equation in t containing two squared terms (which subsequently cancelled) and so showed that t =1.5.

Part (ii) was done well.

Part (iii) was often accompanied by diagrams which showed a horizontal collision. The momentum equation proved awkward, with uncertainty about the direction of motion signs, both before and after coalescence, or not having the "after" mass doubled.

Question No. 5

This question was well answered by most candidates, who used calculus accurately, and used their answers in (i) to complete part (ii) successfully. The commonest mistakes arose from calculator error, and in (ii) substituting t = 2.33-1 = 1.33 into the formula for $x(t)$.

Question No. 6

Though many good solutions were seen, it was clear that the blend of a (t, v) graph and Newton's Second Law proved awkward.

(i) was invariable correct, but (ii) showed weaknesses in resolving parallel and perpendicular to the plane when the context is novel. Incorrect trigonometric ratios were surprisingly common.

The solutions to part (iii) showed the benefit of having a printed answer, which was provided as it was an essential ingredient in the solution to (iv). In such a situation candidates needed to be particularly careful to give an unambiguous proof that 0.5 arose from the sum of the stated frictional force in (ii) and 0.2 x deceleration from *C* to *D.*

The last part was most commonly marred by having an incorrect deceleration, though a few candidates incorrectly calculated the distance between *C* and *D* by using only part of the area beneath the graph.

Question No. 7

Part (i) was often incomplete as candidates were unfamiliar with the term "contact force". The initial part was well answered by many, and candidates who could not find a relevant angle were still able to gain a majority of marks.

In part (ii) many fully correct solutions were found, and the most common error was to omit an answer for the tension.

Part (iii) was challenging for most, setting up and solving two simultaneous $v^2 = u^2 + 2as$ equations being unfamiliar.

4729 Mechanics 2

General Comments:

Candidates once again showed that they were well prepared for the demands of this module, with only a small minority appearing to be lacking in the mechanical ability required. There were many excellent scripts which showed thorough understanding and which scored high marks. A wide variety of scripts, both in terms of presentation and achievement, was seen by examiners. The paper enabled all candidates to show what they could do, whilst also providing a good level of challenge to stretch the most able.

The level of algebraic manipulation needed was higher than in previous years, but the majority were able to cope with this demand. However some candidates seemed to labour over this when there were more efficient techniques available.

On the plus side, candidates are improving on how much detail they give in “show that” questions. And, after many times mentioning the need to be clear in describing a direction of a vector, this is no longer an issue and the majority are now giving a sufficient description to leave us in no doubt at all.

Comments on Individual Questions:

Question No. 1

This question was well done by the majority of candidates, with only a minority not making an attempt to resolve the force *T* in the direction of motion.

Question No. 2

(i) This proved a good source of marks for the majority of candidates. The majority either used standard constant acceleration equations, or quoted and used standard results for time of flight and range of a projectile. The common error was to find only the time for the projectile to reach its maximum height.

(ii) The vast majority of candidates were able to pick up 4 of the 5 marks in this question. A few candidates failed to include the 'below' required for the direction and not all of these were able to pick up the mark by having a suitable diagram but on the whole the need to include 'below' now seems to be recognised.

Question No. 3

(i) Many good solutions were seen to this request, with a few cases where *g* was omitted completely or *km* used with *mg.* Pleasingly most had the force at the peg acting in the correct direction – just a few taking it as vertical. Most realised that the way forward with this question was taking moments about the point *A*, to eliminate the need to take into the forces at that point.

(ii) Candidates often do not use the most efficient method to solve moments questions. The majority of requests can be solved by taking moments once and resolving twice. As moments have already been used in part (i), the best attack for this question is to resolve twice, usually horizontally and vertically. Those who did this were usually the most successful. Candidates who attempted a second moments equation were usually less successful, either omitting the moment of a force or having incorrect distances.

Question No. 4

(i) The majority of candidates performed well on this question, showing a good understanding of the use of power in both situations. Where an error was seen, this was generally to omit the weight component in their solution.

(ii) This part proved more difficult for a significant number of candidates. The main issues were to omit at least one energy term from a conservation of energy equation, to have incorrect signs with the terms also. As in previous years, some candidates duplicated the gravitational potential energy term by including the weight component down the slope in the resistance to motion.

Question No. 5

(i) Candidates generally knew what to do here, and the question was usually done well; a few resolved the weight instead of the force, some could not find the angle, and some gave the answer as 4.9 when the request was to find the magnitude of the force in terms of g.

(ii) Generally well done, with the first 3 marks being earned quickly. The final mark sometimes proved too difficult, mainly for those who were not working in terms of g. The value of r was not always found and when it was, it was not always 3. Formulae such as $a = r\omega^2$ and $R\cos\theta = mr\omega^2$ often written down but candidates then made no attempt to substitute values. A number of candidates produced unconvincing final steps or used statements to 'show' a decimal answer was the same as the AG. Some candidates did realise at the end that earlier working needed to be converted back to g form.

(iii) This proved a difficult request for a significant number of candidates. The problems encountered were basic understanding that this is a new situation compared to earlier and a new normal contact force was required to be calculated. Also many did not realise that the angle that the string makes with the horizontal (or vertical) was required, many using the earlier angle calculated or, in a minority of cases, using 45°.

Question No. 6

(i) This part was often well done. The area of a semicircle was sometimes taken to be πr^2 rather than $\frac{1}{2}\pi r^2$ but most candidates were able to apply the formula to achieve the correct result for the CoM of a semi-circular lamina. Marks sometimes lost due to failure to add 10 for the CoM of the semi-circular lamina from AE. Candidates generally recognised the fact that 'show that' means detail is required.

(ii) This part was much less successful, with many problems down to poor algebra. The correct angle was not always recognised, and a failure to subtract the CoM found in part (i) from 10 were the most common mistakes. The algebraic manipulation was beyond some candidates. Those who fared best had often rearranged to $x + 4r = 10$ before substituting in for x. Many candidates just picked up the final 2 marks for solving but there were also many candidates who omitted this part.

Question No. 7

(i) This question was a good source of marks for many candidates. A few made a sign error in one of the two equations so the directions were not consistent (the direction used as positive was not always clear and diagrams, if shown, frequently failed to indicate the direction of the spheres after collision). Most picked up the mark for solving although there were some who obtained expressions for e in terms of v_A or v_B from which they tried to show $e > 1/8$ without finding, as asked, the speeds of the spheres. The accuracy mark for the speeds was frequently lost because the chosen directions led to a correct velocity for v_A which was not converted to speed.

(ii) Most got the first 3 marks of this part, though some got the masses mixed up, and others failed to follow through their speeds. Quite a lot claimed to get 7/8 when they clearly had not. A small number of candidates (generally who had got in a mess expressing speeds in terms of e) wrote the KE equation in terms of speed, substituted for one and solved, and found e correctly that way. A minority of candidates used an equation, $\frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(u_A - u_B)(1-e^2)$, that is not a requirement for this specification, but is an acceptable method. However, as with all formulae used, they must be quoted correctly.

(iii) Complete solutions to this part were very rare, with a significant number of candidates making no attempt at all. Those who did make an attempt usually gained the first 2 marks for a conservation of momentum equation and a restitution equation. Not all of those realised that they then had to use the fact that, after the second collision, *B* needed to move faster than *A*, and very few realised that they had to use the fact that the velocity of *C* remained positive.

4730 Mechanics 3

General Comments:

Candidates were usually able to make some progress on each of the questions, although there were elements that proved challenging andvery few gaining more than about 65 of the 72 marks available. The question on motion in a vertical circle and the question on the equilibrium of rigid bodies were sometimes omitted entirely. A number of candidates failed to complete the question paper; in many of these cases there was evidence that candidates had spent a long time on one or more of the earlier questions, often making repeated attempts, or had written very long solutions, often with unnecessary extra lines of working.

Many candidates presented their work well, with clear and understandable working. However there were a considerable number of scripts where writing was difficult to read; an examiner can only give credit where work can be read and understood. Candidates who made errors and had to make a second attempt at a question usually made appropriate use of additional paper.

Some candidates did not answer questions fully. For example, in Question 2(ii) some candidates only gave one of the required distances below *O*, and in Question 4(i) the speed of *A* or the direction of motion of *A* or the coefficient of restitution was sometimes omitted from otherwise good solutions.

Comments on Individual Questions:

Question No. 1

Most candidates correctly applied the cosine rule, with or without a diagram, to find the angle between the directions of motion before and after the impulse acts. Most then went on to use the sine rule to find the angle between the direction in which the impulse acts and the initial direction of motion of the particle, though many wrongly gave the answer as 55.8°.

A small number of candidates drew an incorrect triangle of impulse / momentum. More candidates than usual attempted the question by looking at the motion parallel and perpendicular to the original direction of motion of the particle. A considerable number of these did no more than write down the initial equations.

Question No. 2

(i) Almost all candidates were able to do this part correctly. Some candidates went through an unnecessary extra step by first finding the speed of the particle when it had fallen 1.2 m from *O* and the string was not stretched. A small number of candidates ignored the initial speed of the particle, and a few others made sign errors in the energy equation and so were unable to prove the result legitimately.

(ii) This question proved difficult, and candidates tackled it in a number of different ways. The approach which was most often successful was to equate the initial kinetic energy of the particle at *O* with the total energy in the spring and the string at a distance *x* m below *O* less the loss in potential energy. This approach leads directly to a quadratic equation which has the required distances as its roots. Other approaches were to define *x* as the extension of the string, or the reduction in length of the spring or as the length of the spring when compressed, and then find and solve a quadratic equation. Some candidates used the total energy of the system when the particle was at *T* as their starting point but they often missed one or other of the terms needed in forming the energy equation. There were some good attempts at this question where the candidate then went on to discount one of the solutions, and only gave one length as the answer.

It is also possible to show that, after becoming attached to the spring, the particle performs simple harmonic motion, and then find the required distances. This approach was only rarely seen.

Question No. 3

(i) This was usually correct, though 12 (m s $^{-1}$) was a common wrong answer.

(ii) This part was often done completely correctly. However, some candidates thought the integration resulted in a logarithmic function, while others made numerical slips in the integration; some candidates made an error in finding the constant of integration and others had difficulty converting their expression for *t* to an expression for *v*. Some candidates gave their answer as a quadratic expression in *t*; any correct answer in the form $v = 48 - A\left(B - \frac{t}{C}\right)^2$ was also accepted.

(iii) Candidates with the wrong answer to part **(i)** were often able to gain a mark for writing their *v* as $\frac{\mathrm{d}x}{\mathrm{d}t}$, and another for showing the need to convert the limits for *v* given in the question to limits for *x*. The most common error on this part by candidates using a correct expression for *v* from part **(ii)** was to use the given limits of 12 and 32 as limits for *t*.

Question No. 4

(i) There were many correct and concise answers to this part. There were also many answers where candidates wrote down everything they could about the situation, including considering momentum perpendicular to the line of centres, and were then not able to see a way through the problem. There were quite a number of cases of candidates omitting to find either the speed of *A*, or its direction of motion, or the coefficient of restitution, in an otherwise correct solution.

(ii) Some candidate answered this question accurately and briefly by stating that the (component of) speed of both *A* and *B* perpendicular to the line of centres remained equal. Other candidates filled up all the answer space, often with unnecessary explanations that there would be a collision if *U* is large enough – which was stated in the question. Many of these candidates rescued their answer by including the correct statement, in some form.

(iii) The most common successful approach to this part was to write down an equation for the conservation of momentum parallel to *XY* and another using Newton's experimental law, and then use the fact that as long a *U* is not greater than the speed of *B* there will be no second collision. Many candidates were unsuccessful on this part because of sign errors or using an incorrect mass in the momentum equation. A small number of candidates wrongly stated that *U* needed to be strictly less than $\frac{1}{15}$.

Question No. 5

(i) There were two main approaches to this part. Some candidates equated the total potential energy of *A* and *B* in the initial position with their total energy when angle *AOT* is $\left(\frac{1}{6}\pi + \theta\right)$ radians; other candidates considered the increase in potential energy of *B*, the decrease in potential energy of *A* and the increase in kinetic energy of both particles. Either approach, explained clearly, could gain full marks. However, some candidates did not show enough working to convince examiners that they deserved full marks.

(ii) Most candidates realised that they needed to use Newton's Second Law for particle *A* when angle *TOA* is $\frac{1}{3}\pi$, but only a minority of candidates arrived at the correct answer. The most common error was to have the wrong mass in one or more of the terms in the Newton's Second Law equation; generally to use *m* but 2*m* and 5*m* were also seen. The other common error was not to be able to realise that when particle *B* passes through point *T* the angle θ is 30°.

Question No. 6

(i) This part was done well by most candidates. However, some candidates worked out the angle at *P* in degrees, rather than using an exact value when taking moments, and so dropped a mark. Others made a slip and arrived at $\frac{1}{2}\lambda U$ for the force at *Q*.

(ii) This part was omitted by quite a number of candidates, and many others wrote down an equation or two, often correctly, but then made no further progress. There were also a good number of perfectly correct solutions; some of them were very efficient. This question needs to be tackled by resolving vertically and horizontally for the whole system, to get two equations in terms of *P*, *Q*, *W* and *U*, though many candidates used the results from part **(i)** to avoid using *P* and *Q*. *P* and *Q* can be eliminated, and *U* written as *kW* to give two equation in *k* and λ which can be solved simultaneously. Some candidates found an efficient short cut when they realised that the $Q \sin 45°$ term in one equation was equal to the $Q \cos 45°$ term in the other. This led them to find the value of *k* quickly but some then failed to find the value of λ.

Taking moments about *A* for the whole system does not work – the terms cancel out since this has effectively been used to do part **(i)**. Taking moments about *B* or *C* for the whole system is extremely complicated, because the directions of the forces at *P* and *Q* are difficult to manage. Working on either rod *AB* or rod *AC* means introducing components of the force at *A*; marks were only allocated to this approach when these components were eliminated.

Question No. 7

(i) Most candidates who attempted this question were able to show that, assuming the string made a small angle with the vertical throughout its motion, then the motion of *P* initially is simple harmonic. Just a few candidates missed out the minus sign or the length 0.8 m. Most went on successfully to find the period of the motion. Only a fairly small proportion of candidates actually 'proved' that the motion was simple harmonic by also showing that the maximum angle the string makes with the vertical is small. Some of these candidates showed the angle was small at the start of their answer while others showed this when they needed to replace sin θ by θ in their proof. Some candidates only realised the need to show this when they needed to find the amplitude in part **(ii)** and, in this case, credit was awarded in part **(i)** for the work shown in part **(ii)**.

(ii) Candidates with the correct amplitude were usually able to find the times taken for the string to make an angle of 5° with the vertical, though not all found the time that elapsed between the two occasions. Candidates with the wrong amplitude were allowed partial credit for a correct method to find a time, and for a correct method to find the linear speed of *P*.

(iii) A small number of candidates answered this very well, by finding the angle made with the vertical by the string when *P* at its highest point to the right of *Q* and explaining that this angle is too big for the approximation of sin θ by θ to be valid. Many candidates gained partial credit for an answer stating that the angle was too big without showing any detailed working. Other candidates gave wrong answers referring to the string becoming slack, *P* moving in a circle or other forces being involved to suggest that the motion is not simple harmonic.

4731 Mechanics 4

General Comments:

The work on this unit was generally of a high standard. Many of the candidates were very competent and demonstrated a sound understanding of the principles of mechanics covered in this module. However, a small number of candidates struggled with the majority of the paper and were not able to apply principles appropriate to the situations. Candidates seemed to be particularly confident this series on solving problems regarding relative velocity, applying the principle of conservation of mechanical energy and using energy to investigate stability of equilibrium. The topics which provided more of a challenge this series included finding the *y*-coordinate of the centre of mass of a solid of revolution (when the corresponding curve was rotated about the *y*-axis), applying the principal of conservation of angular momentum and finding the force exerted on a body by the axis of rotation. Candidates appeared to have sufficient time to complete the paper.

The standards of presentation and communication were high, though some candidates failed to include necessary detail when establishing given answers.

Comments on Individual Questions:

Question No. 1

Part (i) was answered extremely well with the majority of candidates correctly applying the standard formula for the period of small oscillations of a compound pendulum. The main issue came in the calculation of the moment of inertia about the horizontal axis when a number of candidates incorrectly added $2M(2a-ka)^2$ to $\frac{1}{3}(2M)(2a)^2$. Furthermore, a number of candidates used incorrect values for the mass and length of the rod. In part (ii) the vast majority understood the need to differentiate, with respect to *k*, the expression $\frac{4+3k^2}{k}$ and to then set this answer equal to zero to find the value of k^2 for which the period of oscillations would be least. A number of candidates made this part significantly more demanding for themselves by attempting to differentiate the whole expression for T. Some candidates did not read this part carefully and gave the value for k without ever stating the value for k^2.

Question No. 2

In recent years candidates have found the questions on relative velocity to be rather demanding and so it was encouraging that this particular question on interception was answered extremely well. In part (i) most found the bearing correctly by applying the sine rule using an angle of 85° (which came from $180-60-35$) and then subtracting the value obtained from 60 to get the correct bearing of 024°. In part (ii) the majority correctly used either the cosine or sine rule to find the distance travelled by *S*.

Question No. 3

The vast majority of candidates correctly derived the given result for the first derivative of the total potential energy of the system in part (i). Nearly all candidates correctly stated the gravitational potential energy of the rod as $-2mga\sin\theta$ but a minority did not read the question carefully and took the fixed horizontal rail as the reference level for gravitational potential energy rather than the fixed horizontal axis at *A*. Nearly all candidates correctly dealt with the extension in the light elastic string as $2a+4a\sin\theta$ and went on to correctly apply the formula for the elastic potential energy of a string. In part (ii) many candidates found the level of mathematical sophistication and corresponding algebra required to be far too demanding. However, the majority of candidates started this part correctly by setting the given result from part (i) equal to zero and stating that for $\lambda > \frac{1}{12}$ there would be positions of equilibrium at both $\theta = \frac{\pi}{2}$ and $\sin\theta = \frac{1-4\lambda}{8\lambda}$. Most candidates then correctly differentiated the given result from part (i) and then proceeded to substitute $\theta = \frac{\pi}{2}$ into their expression for the second derivative. If done correctly candidates should have achieved $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -2mga(12\lambda-1)$ and then argued that if $\lambda > \frac{1}{12}$ then $(12\lambda-1)>0$ which gives $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}<0$ therefore giving an unstable position of equilibrium. For the other position at $\sin\theta = \frac{1-4\lambda}{8\lambda}$ the second derivative simplified quite easily to $16\lambda mga\cos^2\theta$ which is clearly positive for all values of θ so giving a stable position. However, many candidates did not realise that the second derivative could be simplified this easily (as they failed to note that at this position $4\lambda(1+2\sin\theta)-1=0$) and so found it much harder to argue that their expression in terms of λ was either positive or negative when $\lambda > \frac{1}{12}$. Candidates struggled with part (iii) as many simply stated that the position would be either stable or unstable without giving sufficient mathematical detail as to why. It was expected that candidates would realise that when $\lambda < \frac{1}{12}$ there is only one position of equilibrium at $\theta = \frac{\pi}{2}$ so therefore $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -2mga(12\lambda-1)>0$ and so giving a single stable position.

Question No. 4

This question more than any highlighted the need for candidates to read the question carefully. Part (i), which was a given answer (as it was felt that this would benefit the candidates in answering the second part), clearly stated that the solid of revolution was formed by rotating the curve about the *y*-axis and so the volume should easily have been found by evaluating either $\pi\int_0^{\ln 2}\left(4^2-\left(\mathrm{e}^{2y}\right)^2\right)\mathrm{d}y$ or by considering $\pi(4)^2(\ln 2)-\pi\int_0^{\ln 2}\left(\mathrm{e}^{2y}\right)^2\mathrm{d}y$. However, too many candidates found the volume of the solid formed when the curve was rotated about the *x*-axis and so spent many pages attempting to evaluate the incorrect integral $\pi\int_1^4\left(\frac{1}{2}\ln x\right)^2\mathrm{d}x$. A number of candidates surprisingly used a shell method to find the required volume and applied the less well known (but equally correct) formula $2\pi\int_a^b x\mathrm{f}(x)\mathrm{d}x$. However, these candidates then had to spend a considerably amount of time using integration by parts when it came to evaluating the integral $2\pi\int_1^4 x\left(\frac{1}{2}\ln x\right)\mathrm{d}x$.

Part (ii) was the least well answered part-question on the entire paper with many candidates not realising that the exact *y*-coordinate of the centre of mass of the solid was found by considering either $V\bar{y} = \pi \int_0^{\ln 2} 16y - ye^{4y} \mathrm{d}y$ or by evaluating $\frac{\pi}{V}\int_1^4 x\left(\frac{1}{2}\ln x\right)^2 \mathrm{d}x.$ Those that did were usually successful in dealing with the required integration.

Question No. 5

Part (i) was answered extremely well with many candidates correctly deriving the given result for the moment of inertia of the cone about the *y*-axis. The most common error was a failure to apply the parallel axis theorem to the moment of inertia of an elemental disc about its diameter which, if done correctly, would have led to the expression $\frac{1}{4}\rho\pi y^4\delta x + \left(\rho\pi y^2\delta x\right)x^2$. This expression, together with the results $\rho = \frac{3M}{\pi a^2 h}$ and $y = \frac{a}{h}x,$ needed to be integrated (with respect to *x*) between the limits of 0 and *h* to obtain the required result. Candidates found part (ii) demanding with many failing to realise that the parallel axis theorem had to be applied to find the moment of inertia of the cone about the axis of rotation (which was not, as many candidates assumed, the *y*-axis). Even for those candidates who realised this, many then failed to apply this theorem correctly as they mistakenly added, rather than subtracted, $M\left(\frac{3}{4}h\right)^2$ from the result given in part (i). Most candidates, however, did realise that to find the value of *m* the conservation of angular momentum had to be applied. It was slightly worrying that at this level a number of candidates attempted to use an approach based on the conservation of energy.

Question No. 6

Part (i) was answered extremely well with nearly all candidates correctly deriving the moment of inertia of the frame about the axis through *A*.

Part (ii) was tackled with varying degrees of success as although nearly all candidates who attempted this part appreciated the need to apply the conservation of energy many did not calculate the change in potential energy of the frame correctly (as it would seem that many candidates could not deal with the relatively straight-forward task of finding the centre of mass of the given frame). The calculation of the change in kinetic energy was far more successful. The last request of this part was often left blank but many candidates did appreciate that for complete revolutions to occur $\dot{\theta} > 0$ when $\theta = \pi.$ A number of candidates incorrectly stated that $\omega^2 \geq \frac{4g}{a\sqrt{3}}$ or only stated the set of values for ω without ever stating the corresponding values for ω^2.

Candidates found part (iii) demanding and only a few succeeded in getting this part correct. Only a minority of candidates, when trying to find the magnitude of the force acting on the frame at the axis of rotation, derived the correct equations of motion involving the radial and transverse components of the acceleration. Common errors included sign errors, assuming that the angular acceleration was positive at $\theta = \frac{\pi}{6}$, using a mass of *m* rather than 3*m* and using a radius of *a* rather than $\frac{2\sqrt{3}a}{3}$. However, the most common error was in the equation for the radial acceleration in which the majority of candidates used $r\omega^2$ for the angular acceleration when it should have been $r\dot{\theta}^2$. Finally, it was surprising how many candidates attempted to find the angular acceleration of the frame by applying the rotational form of Newton's second law. It should be noted that, in this type of situation, it is much easier, and far less time-consuming for candidates if they simply differentiate the given expression for the angular velocity with respect to time.

4732 Probability & Statistics 1

General Comments

Candidates generally found this paper readily accessible. Most candidates scored well on the standard calculations such as those in questions 1(i) & (iii), 2(ii), 3(i)(b) & (ii) and 6(i) & (ii). A few questions contained relatively non-standard requests (e.g. 1(iv), 5(iii), 6(iii), 7(i) & (ii) and 9(ii)) and some candidates could not handle the slightly different approaches that were needed. In particular, in question 6(iii) there appeared to be a number of candidates that did not know where to start. Questions similar to 9(ii) have been asked several times in the past, but very few candidates seemed to identify what was actually being asked for.

Looking towards the new syllabus

The statistics sections of the examinations for the new syllabus will require less calculation and more interpretation than the current syllabus. For example, candidates will not be able to rely on scoring easy marks for routine calculations, such as those for a regression line or correlation coefficient, but they will need to be able to interpret a value of r in context. In the binomial distribution, the new syllabus will explicitly require candidates to understand the difference between conditions and assumptions. Using question 8 in this paper as an example, the question makes it clear that two conditions for the binomial distribution are already satisfied (a fixed number of trials and two possible outcomes to each trial). So candidates who give these as the required assumptions are showing a misunderstanding. The other two conditions (constant probability and independence) are not implicit in the question and so have to be assumed. In answers to question 8, either of these would score the relevant mark, but <u>only if they were given in context</u> (ie, for example, the answer should refer to a "parcel arriving", rather than a "success".)

Answers given in words

Most of the questions that required answers given in words required some understanding and could not be easily answered by rote or with standard responses (1(iv), 5(iii) & (iv)). In these questions, many candidates showed little understanding either of the context or of the principles involved. In question 8(i) many candidates gave a rote answer and lost a mark because they failed to give the answer in context. Centres should note that general conditions, quoted from text books, will not usually score any marks unless they are adapted to the context of the particular question.

Questions in which the answer is given, and the request is "Show that . . ."

Candidates need to be advised to take care to ensure that, when an answer is given in the question, they do not assume this answer in their solution. They also need to be advised that, in such questions, all necessary steps must be shown. See the comments below, on question 3(i)(a).

Rounding

Centres should note the rubric about giving answers correct to three significant figures. A few candidates lost marks by premature rounding (particularly in question 7(ii)) or by giving their answer to fewer than three significant figures without having previously given an exact or a longer version of their answer. It is important to note that although an intermediate answer may be rounded to three significant figures, this rounded version should not be used in subsequent working. The safest approach is to use exact figures (in fraction form) or to keep intermediate answers correct to several more significant figures.

In question 1(iii) it was not uncommon for a candidate to lose an accuracy mark by rounding 132.457 to 133, having first rounded it to 132.5.
If the third significant figure is zero, candidates often omit it. And some candidates think that, for example, 0.92 is actually three significant figures, the "0" being the first significant figure.

Candidates who give more than one solution

Some candidates gave two solutions to a particular question and did not indicate which solution they wished to be marked. Examiners are not required to mark both solutions and choose the best one. They are required to mark just one of the given solutions. Centres should emphasize to candidates that they must make a choice between their attempts and should cross out the solution that is not to be marked.

Use of statistical formulae and tables

The formula booklet, MF1, was useful in questions 1(i) & (iii) and 6(ii). Candidates generally used the formula booklet well. In question 1(i) very few candidates quoted their own (incorrect) formulae for *r* rather than using the one from MF1. A small number of candidates thought that, e.g., $S_{xy} = \Sigma xy$ or $\Sigma x^2 = (\Sigma x)^2$. Some candidates also tried to use formulae involving, for example, $\Sigma(x - \bar{x})^2$ rather than Σx^2. These candidates almost invariably misused the formula and scored no marks. In question 6(ii), Σd^2 was sometimes misinterpreted as $(\Sigma|d|)^2$ or even $(\Sigma d^2)^2$ and the formula was sometimes misquoted as $\frac{6\times\Sigma d^2}{n(n^2-1)}$ or $\frac{1-6\times\Sigma d^2}{n(n^2-1)}$ or $1 - \frac{6\times\Sigma d^2}{n}$ or $1 - \frac{6\times\Sigma d^2}{n^2(n-1)}$, despite the formula being given clearly in MF1.

In question 7 there was some confusion between Σ and E, with some candidates thinking that $\text{Var}(X) = \Sigma x^2 - (\Sigma x)^2$.

In question 8, which involved $B(10, \frac{7}{8})$, a few candidates used interpolation in the binomial table rather than using the formula. These attempts gained no marks.

Use of calculator functions

Increasingly nowadays, calculators can provide answers using statistical functions, binomial functions etc etc, without the need to quote a formula and substitute values into it. The problem here is that if candidates write down their answer with no working, they can only score either full marks or no marks, with no possibility of gaining any credit for partially correct working. In most cases, the use of such functions saves very little time and it is advisable to show working instead. However, if candidates wish to use these functions, they would be well advised to input all the relevant data twice in order to check their answer.

It should also be noted that, without working, even a correct answer is not guaranteed to gain full marks.

Other points

Few candidates appeared to run out of time.

In order to understand more thoroughly the kinds of answers which are acceptable in the examination context, centres should refer to the published mark scheme.

Some candidates ran out of space and continued on the back page, or in a separate answer booklet. This is obviously quite acceptable, but centres should emphasise the need for candidates to give a clear indication of the fact that they have written further working on another page.

Comments on Individual Questions

1)	(i)	This was well answered, with a very small minority of candidates making arithmetical errors or one the errors mentioned above.
	(ii)	Almost all candidates answered this question correctly.
	(iii)	As for part (i), this was well answered, with a very small minority of candidates making arithmetical errors or one of the errors mentioned above.
	(iv)	Many candidates gave the correct answer: *x* is controlled. A few used words such as "fixed" or "constant" which are incorrect. Some said that *x* is the independent variable, which was accepted. However, a good number of candidates gave inadequate answers such as "*y* is dependent on *x*" or "*x* is not dependent on *y*". Others gave quite specious answers. For example, although the question asked about estimating a value of *x* from a given value of *y*, some candidates wrote that the reason for using the *y* on *x* line is that we are estimating a value of *y* from a given value of *x*. There were a few irrelevant answers such as those that referred to positive correlation.
2)	(i)	Some candidates omitted some probabilities or some labels. A few gave extra branches, with probabilities on them.
	(ii)	Many candidates answered this question correctly. A few omitted the probability of succeeding on the first attempt and just found $\frac{1}{5}\times\frac{4}{5}$.Others considered both first and second attempts, but incorrectly just added $\frac{4}{5}+\frac{3}{4}$.
	(iii)	Many candidates gave a correct equation involving *p*, but some were unable to handle the ensuing algebra. Not many used the slightly more efficient method, using 1 – P(three failures). Some correctly saw that they could use their answer to part (ii) as part of the method, but many wrote $\frac{19}{20}+\frac{1}{4}p=\frac{197}{200}$. Others considered only the third attempt, giving $\frac{1}{5}\times\frac{1}{4}\times p=\frac{197}{200}$.
3)	(i)(a)	An incorrect method, that gained no marks was $\frac{\text{Number of discs taken}}{\text{Total number of discs}}=\frac{3}{10}$. The fact that the answer is given in the question was an issue for some candidates. Some lost marks because they did not show enough working. For example, P(RRG or GRG or GGR) = $\frac{1}{10}+\frac{1}{10}+\frac{1}{10}=\frac{3}{10}$ is insufficient working to gain the marks because the answer is given in the question. Some candidates unwittingly gave a circular argument. They used P(X = 1) = $\frac{3}{10}$ to find P(X = 3) = $\frac{1}{6}$, and then used P(X = 1) = 1 - ($\frac{1}{30}+\frac{1}{2}+\frac{1}{6}$). Candidates need to be advised to take care to ensure that, when an answer is given in the question, they do not assume this answer in their solution, and that they show all necessary steps. A few candidates (incorrectly) used the binomial distribution, giving $3\times 0.6\times 0.4^2 = 0.288$. Then they rounded this to 0.3 to match the given answer.

		Most candidates used probabilities, while some used combinations. Either method is acceptable.
	(i)(b)	Most candidates answered this question correctly. A few found P(*X* = 3) incorrectly but used a correct method for E(*X*) and Var(*X*). Others simply ignored P(*X* = 3) and gave (otherwise correct) working using only three values of *X*. Some omitted to subtract $(E(X))^2$ in their calculation of Var(*X*). A few used the binomial formulae *np* and *npq*. Strangely, a few candidates thought that $\text{Var}(X) = \frac{E(X^2)}{(E(X))^2}$.
	(ii)	Some candidates ignored the repeated colours and just found 10!. Others arranged the reds and greens separately, giving 6! × 4!.
4)		Some candidates misread this question to mean "Find the probability . . ." rather than "Find the number of ways . . .". These candidates could gain a maximum of 3 marks altogether for all three parts. The same maximum applied for those who used permutations instead of combinations.
4)	(i)	Most candidates answered this question correctly. A few just found 30!.
	(ii)	A common error was addition of the three correct combinations, instead of multiplication.
	(iii)	Arithmetical errors were common in the otherwise correct, but very long, method of adding six products of combinations. Candidates who used the direct method (${}^{14}C_1 \times {}^{16}C_5$) were more likely to obtain the correct answer. Some candidates, incorrectly, found ${}^{14}C_1 \times {}^{30}C_5$ or ${}^{14}C_1 \times {}^{29}C_5$. Others added ${}^{14}C_1 + {}^{16}C_5$.
5)		In this question some tolerance was allowed in reading the graph, but a few candidates lost marks through misreading the scale on either or both axes.
5)	(i)	A few candidates gave the answer 600.
	(ii)	Most candidates answered this question correctly. A few read the graph from 106 instead of from 424.
	(iii)	Many candidates showed that they did not really understand the nature of a cumulative frequency graph. Some stated that, since the graph goes up to 60, the highest mark was 60. Some recognised that, because the data is grouped, the highest mark cannot be precisely identified, but some of these went on to say that the highest mark was in the 55 - 59 class. Some candidates recognised that the fact that the curve became flat at about 54.5 meant that no marks were higher than this, but many of these went on to say that the highest mark was 54, (or 54.5, or 55). A few candidates said that the highest mark was between 50 and 59. Some said that there was only one highest mark, so the teacher could not be correct in saying that it was 54 or 55. Most of these candidates went on to say that the highest marks was 54.5.
	(iv)	Most students gave the correct answer of 25 - 29, generally with a correct reason such as that the graph is steepest here or that the increase in cumulative frequency is greatest here. A few gave an incorrect reason, such as "The cumulative frequency is greatest in this class". Some candidates thought that the mode was where the mark (rather than the frequency) was highest, and so gave the answer 55 - 59. A few candidates found the class which contained the median, rather than finding the modal class.
6)	(i)	A few candidates used the scores instead of ranks in this part and part (ii).
	(ii)	This question was well answered, with only a few candidates making errors such

		as those described above.
	(iii)	This question was found difficult by most candidates. A common error was to find P(Judges both gave same ranks) = $(\frac{1}{5!})^2$. Other candidates appreciated that it was necessary for one Judge's ranks to be the same as the other's except for one adjacent pair being swapped. This gave $\frac{1}{5!}$. But many candidates thought that they had to do the same for the other judge and hence arrived at the incorrect $(\frac{1}{5!})^2$ or $2\times\frac{1}{5!}$. Only a few found how many arrangement of one judge's ranks would give the correct value of Σd^2 (ie 4 arrangements) leading correctly to $\frac{4}{5!}$, or incorrectly to $\frac{4}{5!^2}$. Some candidates then doubled this, because there were two judges.
7)	(i)	A common error was to omit to square 5.8 or 52.3 or both. A few candidates confused Σ with E and thought that Var(X) = $\Sigma x^2 - (\Sigma x)^2$. Many others thought that $\Sigma w^2 = (\Sigma w)^2 = (75 \times 52.3)^2$.
	(ii)	Many candidates found the mean and standard deviation of the second sample of stones alone. Some found the unweighted mean of the two means.
8)	(i)	Some candidates omitted the parameters. Many gave assumptions quoted from text books, without context. Some gave conditions which were already given or implicit in the question, for example "Assume that all 10 parcels are posted." or "Assume that each parcel either arrives or does not arrive." Some attempted to give a correct assumption, but worded it incorrectly. For example "The probability that a parcel arrives is independent of other parcels," rather than, for example "Whether or not a parcel arrives is independent of whether or not other parcels arrive."
	(ii)(a)	Most candidates answered this question correctly.
	(ii)(b)	Some candidates found P(X = 9) or P(X = 10) rather than the sum of these. Others thought that P($X \geq 9$) = 1 – P(X = 9) or even 1 – P(X = 10). Some candidates found the correct answer, but then subtracted this from 1.
	(iii)	The majority of candidates recognised the need to use their answer from part (ii)(a), but many found only P(X = 4) or P(X = 5) rather than the sum of these. A surprisingly large number of candidates omitted the binomial coefficient in P(X = 4).
9)		Most candidates recognised that the geometric distribution was needed for parts (i)(a) and (i)(b). However there was obvious confusion about when a simple power of q is required and when a power of q needs to be multiplied by p. There was also confusion about when to use q^r and when to use $1 - q^r$.
	(i)(a)	This part was answered correctly by most candidates.
	(i)(b)	This part was answered correctly by many candidates, although some multiplied 0.8^4 (or 0.8^3) by 0.2. Sadly, many candidates appeared to be unfamiliar with the short cut method, and used the long method. Some of these added the correct four terms but did not subtract from 1. Others subtracted from 1 but omitted one of the four relevant terms. Some candidates found $(0.8)^3$ instead of $(0.8)^4$. A significant number of candidates thought that P($X \geq 4$) = 1 – P(X = 4).
	(ii)	Questions similar to this one have appeared on a number of past papers, so it was disappointing to find that many candidates did not appreciate what was actually being asked, and so found P(5 vouchers in 10 chocolate bars). As in question 8(iii), a large number of candidates omitted the binomial coefficient. A significant number of candidates began correctly by finding ${}^9C_4 \times 0.8^5 \times 0.2^4$ but then, instead of multiplying by 0.2, multiplied by $0.8^9 \times 0.2$.

4733 Probability & Statistics 2

General Comments:

Many candidates answered most of the paper confidently but were often weak over detail. As usual many answered a question they had seen before rather than the question in front of them; the ability to answer questions that have not been asked before is a key test of a grade A or A* candidate.

Over the lifetime of this specification there has been a pleasing steady increase in the proportion of candidates who handle details such as continuity corrections accurately.

There has also been a steady increase in the number of candidates who obtain probabilities directly from a calculator. Such answers can generally score full marks if they are correct, but if not they will usually not gain even the method marks unless sufficient working is seen. This applies even if the answer is recognisable as likely to have come from a standard mistake such as omission of the continuity correction. Calculator syntax such as “cdfnorm” does not qualify as correct working. Candidates should be encouraged to follow good practice in presenting their solutions in proper mathematical terms; the use of the phi notation is strongly recommended.

Comments on Individual Questions:

Question No.

1(i) The question asked for an explanation of why a method was unsuitable. The following common answers are inadequate for the reasons stated:

- “The method is not random”: some non-random methods (for instance, a systematic sample with non-random choice of the starting point) can be perfectly adequate.
- “The sample is not representative”: simple random samples selected without bias may not be representative purely by chance.
- “Not everyone would be able to reply”: *any* sampling method involves not getting replies from all the population.

Some sort of reason for bias was required, such as “those attending the meeting may have stronger opinions”. In fact the main problem with any method that asks respondents to return a questionnaire is always that of the “self-selecting sample” (those with stronger opinions are more likely to reply), but few candidates focussed on this problem.

1(ii) Candidates were required to explain a method involving random numbers, so those who suggested putting names into a hat did not gain full marks. Some candidates said “number the parents randomly”, which is not an appropriate method unless they are then *sorted* by those random numbers. Some demonstrated a lack of understanding of random numbers by saying “put the numbers into a random number generator”. As in previous years, candidates had to refer to “ignoring repeats” or “ignoring numbers outside the range” in order to score full marks.

2 This question on finding the mean and variance of a normal distribution began easily, as the probabilities given involved no difficult choice of signs. Most candidates got as far as finding the mean (it is not actually necessary to find the variance) but there were frequent sign mistakes in finding the value of a. Many candidates were inaccurate in their working; premature rounding often lost marks by leading to final answers outside the allowed tolerances.

3 A routine hypothesis test for the parameter of a Poisson distribution was often poorly handled. As usual, many who used a probability method found $P(\leq 23)$ or P(= 23) rather than the required $P(\geq 23)$.

In recent years there have been more candidates whose solutions represented a confusion of the probability method and the critical region method. There seemed to be a good deal of "hedging of bets". Some stated a critical value, but often did not make it clear whether this value was or was not included in the critical region. It was often hard to follow the arguments of such candidates, and as communication is a key feature of hypothesis testing, such candidates lost several marks.

As usual, it was necessary to give the final conclusion in context (mentioning "the average number of errors made" or equivalent) and without being over-assertive, so a conclusion such as "Reject H_0. The number of errors made by the new team is greater than 6.3" did not score the final mark.

4(i) A standard normal hypothesis test question. Common mistakes included: not finding an unbiased variance estimate, or the omission of a factor of $\sqrt{36}$ in standardising. Wrong hypotheses were rarer this year. As usual with tests where the outcome is not to reject H_0, the final conclusion needed to be stated as a double negative, for example "there is insufficient evidence that the mean pH is not 6.3"; it is wrong to say "there is significant evidence that the mean pH is 6.3".

4(ii) This question was poorly answered. For a start, many candidates didn't answer this question but their own variation of the question; the question did *not* ask "<u>do you need to</u> use the CLT?", or "<u>can</u> you use the CLT?" but "<u>where</u> is the CLT used?" Answers such as "Yes because we are not told the parent distribution" were disappointingly common.

The central limit theorem appeared to be a widely misunderstood topic. It is *not* a statement about dividing the variance by *n*; that result is true for any distribution. Many candidates seem to believe that the CLT turns any distribution into an approximately normal one; "we are not told that the distribution is normal but the CLT allows us to assume that it is" was a common wrong response. It is important for candidates to realise that the CLT refers to *two different* distributions: the "parent" distribution, and the distribution of the sample mean (not "the sample", either). The statement is: "regardless of the parent distribution, the distribution of the sample mean is approximately normal for a large enough sample". The key words are "sample mean". A correct answer would, therefore, have been, "in assuming that the sample mean is approximately normally distributed".

5(i) Generally well done, although the conditions for the Poisson approximation to the binomial were often poorly stated; "$np = 2 < 5$" was often seen, but "$n > 50$", or "n is large" was also needed.

5(ii) Most candidates correctly used N(72, 36.97) but many then chose the wrong tail or the wrong *z*-value or both. A large number did not deal correctly with the continuity correction and/or the need to round up 89.56 (or 89.06) to the next whole number, 90; answers of 89.6 or 89 were common, and there was much inaccurate numerical work.

6(i) There were many poor answers in stating the modelling assumptions for a Poisson distribution. "Randomly" was stated in the question and is therefore not an extra assumption; "singly" is as usual irrelevant. Many answers used the standard words but made no sense, suggesting that their users had no idea of what they meant — for example, "an article must occur at constant average rate". Again as usual, there were many who thought that "the probability of any article being received must be constant", regardless of the fact that this is (a) a binomial condition rather than a Poisson one, and (b) meaningless. Likewise, "the probability of an article being received is independent" is strictly incorrect; independence refers to events and not to probabilities. A simple correct answer was that articles should be received at a constant average rate throughout the week and independently of one another.

6(ii) Generally very well done.

6(iii) A pleasing number of candidates worked this correctly. However, some were unable to deal with the algebraic manipulation, whilst others ignored the requirement to produce an algebraic solution by omitting working and using tables or a calculator for part or all of their solution.

6(iv) Generally very well done. A large proportion of candidates got the continuity correction right.

7(i) For most this was straightforward, but some attempted to find the median. The misread $x(16 - x)^2$ for $x(16 - x^2)$ was seen several times.

7(ii)(a) Most correctly obtained the given equation; it was necessary to show the limits of 0 and q (or q and 4 if equated to ¼).

7(ii)(b) Almost everyone easily recognised a quadratic in q^2, but candidates at this level were expected to acknowledge four solutions (two of them negative) and then to finish with an exact solution of $2\sqrt{2}$ or $\sqrt{8}$ only. Those who went straight from the equation to 2.828 scored only 1 mark out of 3.

8(i) It was necessary to focus on nq and quote its value as 3, which is less than 5 (or to make an equivalent statement about both n and p: "p not close to ½" is not enough as it depends also in the value of n). The wrong condition $npq < 5$ was quite often used and this did not score the mark.

8(ii) Solutions to this question were often too casual. Many were content to find P(= 60) and state that this was less than 0.05, but it was also necessary to show that $\mathrm{P}(\geq 59)$ is greater than 0.05. Use of the Poisson distribution in this and subsequent parts of the question was possible, though unnecessary.

8(iii) This was found surprisingly hard. Few realised that in order to make a Type I error the value of p had to equal 0.95 and nothing else. Answers such as $p < 0.95$ or $p > 0.95$ were common, but so were answers such as $X = 60$. Candidates seemed to be confused about what p represented. A more understandable common wrong answer to the probability of a Type I error was 0.05.

8(iv) There was a clear-cut division between those candidates who could answer this question confidently and accurately, and those that struggled to make any progress. Some tried to use a normal distribution, and a common wrong solution involved comparing p^{60} with 0.6 instead of 0.4.

4734 Probability & Statistics 3

General Comments

There were 354 candidates, 25% more than last year. As usual, many produced extremely good scripts. There was no evidence of candidates running out of time.

The standard of the stated hypotheses has improved, but there are still candidates who do not make it clear that the hypotheses are about the population from which the sample was drawn. This is best done by using standard symbols e.g. μ. The standard of final conclusions was very good. There were few over-assertive conclusions.

The standard of presentation of many scripts was lamentable. Candidates should be aware that if their written answers cannot be read, they will not be given any marks for them. Also, many candidates lost accuracy marks because they misread their own figures.

Comments on Individual Questions:

Question No.

1(i) Many gained full marks. Those who did not usually used the *t*-distribution and/or did not use the given standard deviation.

1(ii) Roughly half of the candidates gained this mark. The others should have realised that they should refer to a large number of confidence intervals, not just this one.

2 Most candidates gained full marks. A few had incorrect critical values. Others did not pool the samples.

3 Many candidates scored full marks. Those who did not usually had an incorrect critical value. Others had the hypotheses the wrong way round. Some did not use Yates' correction or used it incorrectly.

4(i) Almost all candidates answered correctly.

4(ii) Almost all the candidates knew the correct reason.

4(iii) Many found this question difficult. Despite being told that *Z* did not follow a Poisson distribution, many still used Po(23). Others used N(23,107).

5 Many candidates gained full marks. Common mistakes included incorrect hypotheses, or an incorrect critical value.

6(i) Almost all the candidates answered this question correctly.

6(ii) Most of the candidates gained full marks.

6(iii) Many gained full marks, but classes were not combined by a surprisingly large number of candidates. Many did not use the correct degree of freedom.

7(i) Almost all the candidates answered correctly.

7(ii) Most gained full marks. Some used a lower limit of 0 in the first integral.

7(iii) Almost all the candidates gained the first mark, and most of these went on to score the second. Many simply substituted 2.41 at this point and gained no more marks. However, there were some very good solutions particularly from those who realised that f(x) in the correct range could be written as $\frac{3}{16}(x-4)^2$. This led to the correct exact solution $4+\sqrt[3]{-4}$. Various iterations and Newton-Raphson produced correct solutions. Those who used a GC and gave the answer to more than three significant figures were also allowed full marks.

8 Most candidates scored at least 9 out of 10, some losing the final mark for the assumption. A few did not use a paired test. .

9 This was a very difficult question. Many did not realise that the pdf of U was $\frac{1}{f}$. Some that did, integrated wrt to f rather than U. Most scored the second mark. Very few scored the third mark. Most scored the fourth mark for attempting differentiation, sometimes of very strange functions. Many gave the final range the wrong way round.

4735 Probability & Statistics 4

General Comments:

There were 79 candidates, similar to recent years. As usual, many produced extremely good scripts. There was no evidence of candidates running out of time.

Comments on Individual Questions:

Question No.

1 Many candidates gained full marks. Those who did not usually lost marks for considering only one tail. Others looked for 0.0025 rather than the correct 0.025. Some considered N(15,7.5).

2(i) Most candidates gained full marks. Those who did not usually assumed that U and V were independent.

2(ii) Most candidates gained full marks. Those who assumed independence in part (i), usually gained the four method marks.

2(iii) Most gained both marks, but those who had assumed independence ran into difficulty.

3(i) Almost all candidates answered this question correctly.

3(ii) Most candidates gained full marks. A few made errors in the formula for $\mathrm{P}(A \cup B \cup C)$.

3(iii) This was the most difficult question on the paper. Those who drew Venn diagrams and put the correct probabilities in the correct places did better than those who tried to repeat the method in part (ii). There were few fully correct solutions, but many found one of the correct limits.

4(i) Most gained full marks. This indicates that candidates gave the conclusion in a contextualised form which was not over-assertive.

4(ii) Most gained full marks, but many considered the wrong tail. A few did not use the continuity correction.

5(i) Most of the candidates obtained full marks.

5(ii) Many of the candidates obtained full marks. Some used modulus and usually scored one mark. .

5(iii) Most gained full marks, but many made an error when finding $\mathrm{M}'_X(t)$. e^t was missed out, meaning that the second differentiation was made far too easy. Such candidates usually scored two marks.

6(i) Almost all knew what to do, and most answered correctly.

6(ii) Most of the candidates gained full marks. The most common error was not to subtract $(\mathrm{E}(Z))^2$.

6(iii) Almost always answered correctly.

6(iv) Usually correct, but some found Var Y incorrectly.

7(i) Almost all the candidates answered correctly.

7(ii) Most answered correctly, but there were a few errors in the differentiation.

4736 Decision Mathematics 1

General Comments

Most candidates were able to complete the paper in the time allowed.
The handwriting of some candidates was very difficult to read. Care over writing numbers would have led to fewer mistakes.
Candidates were generally good at indicating when they had continued an answer in an additional answer book and labelling the answer in the additional book with the appropriate question and part number.

Comments on Individual Questions

1)	(i)	Most candidates applied bubble sort correctly, although some showed every comparison or every swap and then did not indicate which row was the end of each pass. Most candidates worked on horizontal lists although vertical working was also accepted when it was evident that this was what a candidate was doing. A common error was to stop a row too early, when the list could be seen to be sorted but the ‘check’ pass had not been carried out, others continued to a sixth pass. Bubble sort can be stopped when either there is a pass in which no values are swapped or no more passes are possible.
	(ii)(a)	Only a few candidates were not able to reverse the process to give at least one of the possible preceding lists. To get the value 27 in the first position after one pass the original list must have started with either 27 40 or with 40 27, in both cases the 40 passes to the end of the list without changing the order of the other three values.
	(ii)(b)	With bubble sort the ‘check’ pass is part of the sort, so there would be two further passes.
2)	(i)	Some candidates told us everything they knew, or thought they knew, about Josh’s graph in this first part. The question had specified that candidates were to answer without doing any calculations and without drawing graphs. Part (a) asked how the given vertex order show that Josh’s graph is not Eulerian – Josh’s graph has a vertex of odd order (5) so the vertices are not all of even order and the graph cannot be Eulerian. Part (b) asked how the given vertex orders show that Josh’s graph is semi-Eulerian – odd order vertices come in pairs and we know that there is one odd order vertex so one of the missing orders must also be odd (and the other even), which means that there is exactly one pair of odd order vertices and hence the graph is semi-Eulerian. Just stating that a semi-Eulerian graph has two odd order vertices was not enough, candidates needed to explain how they knew that Josh’s graph had exactly two odd order vertices.
	(ii)	The easiest way to calculate the missing vertex orders was to use the fact that the graph has 8 arcs to deduce that the sum of the vertex orders is 16 and hence the sum of the missing orders is 3. Since the graph is connected the missing orders cannot be 0 and 3, so they must be 1 and 2 (and there are no other possibilities).
	(iii)	This last part was asking why all graphs with six vertices of orders 5, 4, 2, 2, 2, 1 must have the same structure (must be isomorphic). Candidates who started from ‘it is a simply connected semi-Eulerian graph with six vertices and eight arcs’ and tried to deduce that these must be the vertex orders were doomed to fail; there are other possible vertex orders that fit this description (e.g. 4, 3, 3, 2, 2, 2). In general

		there will be multiple graphs with different structures that have the same set of vertex orders, in this particular case there is only one possible structure, but showing this involves considering which vertices can be connected to which other vertices.
3)	(i)	Most candidates were able to trace through the algorithm, although some wrote all their working on one row, instead of using one row for each line when a value was updated. A few made numerical errors, often in calculating the new value of *A* or *B*.
	(ii)	Many correct answers. Some candidates recorded *M* changing to 5 in the same line 50 row as where they had recorded *M* becoming 10, instead of having a new line 50 row following the line 70 row.
	(iii)	Some candidates incorrectly stated that repeatedly dividing by 2 would never give an odd number. Those who realised that *M* would eventually become 1 (sometimes from having tried out a specific case, such as $N = 8$) often got $P = x$ (although sometimes this value was out by 1) and hence the final values $B = 0$ and $A = 2^x$ (or the original *N* value).
4)	(i)	Most candidates identified the odd vertices correctly.
	(ii)	Candidates usually calculated the journey times for *A* – *D* – *F* and *A* – *E* – *F* but did not always answer the question asked, which was to find the shortest travel time (29) not to state the route.
	(iii)	Dijkstra's algorithm was usually done correctly apart from candidates whose arithmetic was faulty. A common error was to write down all the distances to a vertex instead of only updating the temporary label when a new distance is shorter than the current best temporary label.
	(iv)	The sum of all the values in the table was given as 444, but each arc appears twice so the sum of the arc weights is 222. Most candidates said that $BE = 24$ needed to be repeated but did not say that this used the arcs *BC* and *CE*. This meant that the vertex order of *C* had to be increased from 6 to 8, but each time the route passes through *C* it uses up two of these arcs, so *C* is passed through 4 times and not 8 as several candidates suggested.
5	(i)	Most candidates were able to apply the nearest neighbour method correctly, a few returned to *P* before every vertex had been visited. Nearest neighbour uses the shortest arc from the current vertex to a vertex that has not yet been visited, until all vertices have been visited and the route returns to the start. In this case the route that was obtained by returning to *P* midway through the route was in fact longer than the nearest neighbour route.
	(ii)	Candidates could usually find one of the two possible improved routes and its weight.
	(iii)	Many correct answers were seen although some candidates did not show that they had considered the arcs in increasing order of weight and some only listed the arcs that had been chosen.
	(iv)	Many candidates realised that *k* had to be greater than 9, but often the explanation assumed that the tree was being built using Kruskal's algorithm and the possibility

		of a different minimum spanning tree using Prim's algorithm was only occasionally considered. Unless the totals are considered very carefully, if a tree is formed using Prim's algorithm then if M is joined in before L the value of k also needs to be compared with $MN = 6$.
	(v)	The answer $9 < k < 12$ was often seen, though $k < 12$ was also accepted. Sometimes candidates went on to assume that the arc weights had to be integer-valued and gave $k = 10, 11$ as their answer. Although the arc weights in the question were all integer-valued this was not stated as a restriction. A small, but not insignificant, number of candidates correctly found the length of the shortest cycle that does not include LM and the length of the shortest cycle that does include LM and used these to deduce that $35+k < 47$ so $k < 12$.
6	(i)	Most candidates found the coordinates of at least four of the five points of intersection.
	(ii)	Either by evaluating P at each of A, B, C, D and E or by using a sliding profit line, candidates were usually able to identify 25 as the minimum feasible value. Some candidates gave the coordinates of the point where this occurred rather than giving the minimum feasible value and a few calculated the value 25 in their list but stated 26 as the minimum.
	(iii)	Usually done well, apart from candidates who were using the wrong pair of simultaneous equations. A few worked out the coordinates of the new vertex but not the value of the objective at that point. Some candidates assumed that the coordinates had to be integer-valued at this stage.
	(iv)	There were two things to do here: show that (5, 1) is feasible and show that there are other integer-valued feasible points at which the objective is smaller. To show that (5, 1) is feasible involved showing that the coordinates satisfy the constraints, stating that it is to the left and above the point found in part (iii) is not sufficient. To show that (5, 1) is not the minimum required calculating the value of the objective at (5, 1) and also identifying an integer-valued feasible point with a smaller value of P.
7	(i)	Many candidates offered the inequality as either $x + y + z \leq 265$ or $15x + 50y + 200z \leq$ some value, clearly neither of these are consistent with the upper limits of 15, 50 and 200. Other candidates realised that the coefficients represented the amount of space occupied by one item but were not able to progress beyond that. Those who realised that $d = 15a = 50b = 200c$ (rather than $15x$, etc.) could usually achieve the required inequality.
	(ii)	Several candidates realised that the pivot had been the x-column. Some gave z (the next pivot choice rather than the previous one) and a few offered other columns.
	(iii)	The second iteration was often done well, provided the pivot choice was correct and there were not too many arithmetic errors. Candidates should check that their answer has the correct structure (all cells filled in, basis columns, no negative entries in RHS column).
	(iv)(a)	Interpretation of the given final tableau was poor. Even the candidates who wrote

		down the numerical values of *P*, *x*, *y* and *z* correctly did not always interpret the solution in context.
	(iv)(b)	The question had already said that the other constraints took account of the amount of clay required, the potter's time and customer demand. The cost of running the kiln will have been incorporated into the profit equation. Most candidates said that to achieve this profit the potter has to sell all the goods, this was fine. Some candidates realised that a pot could be damaged in the kiln and not be suitable for sale at the full price.

4737 Decision Mathematics 2

General Comments

Most candidates were able to complete the paper in the time allowed.
The handwriting of some candidates was very difficult to read. Care over writing numbers would have led to fewer mistakes.
A few candidates used additional booklets, often only writing a few lines on an additional booklet whilst leaving the additional space on page 12 of the answer book blank. Some candidates used an additional booklet and left the original answer space in the answer book blank. Although examiners will try to understand a candidate's intention, the misuse of the answer book could lead to a loss of marks when the replacement answers are not labelled.

Comments on Individual Questions

1)	(i)	Most candidates were able to draw a correct labelled bipartite graph, some were not sure how to deal with the requirement of two workers for activity W
	(ii)	Most candidates were able to write down the shortest alternating path *C-X-A-W* and use it to write down the incomplete matching with *D* left out. Some candidates gave a longer alternating path and some appeared to be writing arcs rather than an alternating path. A few candidates did not attempt an alternating path and just gave the matchings. Some candidates did not write down the incomplete matching at this stage and just drew a diagram - the question had specifically asked for written allocations.
	(iii)	A few candidates claimed that there were 120 or 60 or 5 possible matchings, but most candidates realised that there were 2. The majority of candidates appreciated that both *A* and *B* would have to do *W*, although some felt that they could swap over *A* and *B* and so thought that there were 4 allocations instead of 2. The explanation of why there are only 2 ways to allocate *C*, *D* and *E* to *X*, *Y* and *Z* needed to be more than just a listing of two possible allocations – this shows that there are 2 but not that there are only 2. There are two possibilities for *C*, if *C*=*X* then (ignoring *A*, *B* and the *W*'s) we get the alternating path *C-X-D-Z-E-Y* which means *C*=*X*, *D*=*Z* and *E*=*Y*, if *C*=*Y* then (ignoring *A*, *B* and the *W*'s) we get the alternating path *C-Y-E-Z-D-X* which means *C*=*Y*, *D*=*X* and *E*=*Z*.
2)	(i)	Candidates who put Tom's choices (the cards in Tom's hand) on rows were usually successful, although some said that Tom should choose card 2 from his hand and card 1 from the table – even though the question said that he could not see the values of the cards on the table. The question asked candidates to 'hence find Tom's play-safe strategy' so it was necessary to explain how the values in the table led to the play-safe, for example by showing the worst outcome for Tom for each of his choices and hence the strategy (card) that gave the maximin for Tom.
	(ii)	Similarly in this part, candidates who put Tom's options on rows were generally more successful than those with Tom's choices on columns. A few candidates did not label the rows and columns and some tried to write the payoffs as algebraic expressions, e.g. $-(A^2)$, instead of numerical values, such as -1 or -16.
	(iii)	Most candidates realised that when Tom held the card numbered 6 this was his play-safe. A few said that when he holds this card he can always get a positive payoff, but did not say which card he should play. Some candidates then assumed that if he did not hold the card numbered 6 he

		must have the cards 2, 3 and 5 or said that he should play the card numbered 1 without any recognition that he may or may not hold this card.
	(iv)	If Tom's play-safe is the card numbered 3 then he could not have the card numbered 6 (from (iii)(a)) and 3 had to be his smallest card (from (iii)(b)), so he must have the cards 3, 4 and 5.
3)	(i)	What was needed here was an explanation of how the values 2, 2, 1, 6 in this row show the current stage and state and how the action value determines the the next stage and state, using specific numerical values, rather than general statements.
	(ii)	Many fully correct answers and a few where candidates had found a maximum path or a minimum path. Occasional numerical slips incurred only a small penalty but failure to actually solve a maximin problem incurred a greater loss of marks. Candidates should show the two values being minimised and their minimum in the working column, for example min(7, 5) = 5. Despite being asked to write down the maximin value and state the maximin route from (0; 0) to (4; 0) some candidates did not give the maximin value and some left (4; 0) off the route.
4)	(i)	Many excellent answers. A few minor numerical slips but most candidates appeared to be doing the right thing. Only a tiny number of candidates changed the problem into a maximisation at the start. Most candidates described the row and column reductions, although sometimes labelling these at the matrix prior to the change being applied rather than the matrix that showed the result of it happening. Most candidates either used the term 'augmented matrix' or described subtracting the minimum uncovered entry from each uncovered value and adding it to each value that was covered twice. A few candidates only added 1 to the entries that were covered twice. The allocation was usually correct, even when there were earlier errors.
	(ii)	Changing N=B (for example to 10) changes the minimum costs, so here the original costs needed to be used rather than the reduced costs. Also, while showing that the given allocation only costs £1000 more than the original solution, this neither shows how it was found nor that it is unique. By considering the minimum cost in each column of the original matrix it becomes apparent that this is the best solution. Considering row minima needed a careful explanation, particularly in respect of Jai and Nina.
	(iii)	Now N=B is not allowed as well as having J=L at no cost. Again, because of these changes the reduced cost matrix is no longer valid and the original costs must be used. The answer hinges on what Nina does, and the fact that using N=C gives the next best solution (not N=G), but that this costs £2000 more than the solution from part (ii), or that this costs £5000 rather than £1000 and that £2000 is saved on using J=L at no cost instead of J=C.
	(iv)	Many answers offered here. Some candidates thought that because N=C costs £5000 the cost of J=C would need to be £5000, but in fact the cost from (iii) is only £2000 more than the cost from (ii) so the cost here needs only to match £16000. This means that the cost here must increase by £2000, which means that J=C must increase from £2000 to £4000 (as the minimum possible value).
5	(i)	The dummy between events (2) and (3) is needed to show that *E* follows both *A* and *B* while *C* and *D* follow from *A* only, without needing *B* to be complete. This dummy is needed to deal with the precedences at event (2)

		The dummy between events (4) and (5) is needed for uniqueness, so that activities *C* and *D* do not both start at event (2) and finish at event (5). Some candidates claimed that the problem was that *C* and *D* have different durations, but the issue is that we cannot represent the arcs using their start and finish events unless the network is built on a simple graph.
	(ii)	This was mostly done well, and even the candidates with errors often only had at most one independent error in each direction. The late times for event (6), and hence for event (3), were frequently miscalculated.
	(iii)	Most candidates were able to state the minimum completion time as 23 hours (a few said days or minutes) and the critical activities as *A*, *D*, *F* and *J*. A few gave the events that defined the critical activities instead of the actual critical activities.
	(iv)	Most candidates were able to show the critical activities *A*, *D*, *F*, *J* on a single row and the majority of candidates put each of the non-critical activities on a separate row, as instructed. Most candidates had the early start and finish times correct, but often at least one of the late times (e.g. the late time of *B* at 8) was wrong. Sometimes it was difficult to see which boxes candidates had chosen, shading was often useful. Most candidates labelled the activities appropriately.
	(v)	Here the minimum time is achieved by having *C* and *D* one after the other with *E* happening alongside them, so *C*, *D* and *E* take 7 hours, after which *F*, *G*, *J* and *K* can happen, one after the other. It was not necessary to delay the start of activity *E*. This gives a completion time of 29 hours.
	(vi)	Now *E* cannot happen alongside *C* and *D*, although *C* can happen in parallel to *D*. This adds another 4 hours to the duration, making a completion time of 33 hours. Several candidates were able to deduce the minimum time of 33 hours for this part, even when they had come unstuck in the earlier parts.
6	(i)	The network has one source, at *A*, and two sinks, at *C* and *E*. Some candidates added a supersink that *C* and *E* were both connected to.
	(ii)	The arc *DG* has a minimum capacity of 3 cm^3s^{-1} and a maximum capacity of 3 cm^3s^{-1}, so the flow from D to G must be 3 cm^3s^{-1}, and cannot be anything else. Some candidates tried to work out excess flows and potential backflows, but for this arc the flow is 3 cm^3s^{-1} and then there is no excess and no potential backflow.
	(iii)	At least 4 cm^3s^{-1} must enter vertex *D* and at most 4 cm^3s^{-1} can leave vertex *D*, so vertex *D* has 4 cm^3s^{-1} flowing through it, with *AD* at its minimum of 4 cm^3s^{-1} and *DC* (and *DG*) at the maximum flow of 1 cm^3s^{-1} (and 3 cm^3s^{-1}). The best answers summarised this information at the end by stating *AD* = 4, *DC* = 1.
	(iv)	A similar argument at vertex *B* shows that *AB* must be at its maximum capacity of 3 cm^3s^{-1} and *BC*, *BE* must be at their minimum capacities of 1 cm^3s^{-1} and 3 cm^3s^{-1}, respectively. Again, the best answers finished by summarising these flows.
	(v)	The question said to consider the flow through *G*, *H* and *F*. The candidates who did this, in this order, were usually successful. At *G*: *DG* = 3 in and *GF* + *GH* = 1 + *GH* out, so *GH* must be 2 cm^3s^{-1}. Now at *H*: *GH* = 2 in and *HF* + *HE* = ($\geq$ 1) + ($\leq$ 1) out, so *HF* must be at least 1 cm^3s^{-1}. Finally at *F*: *GF* + *HF* = 1 + ($\geq$ 1) in and *FC* + *FE* = ($\leq$ 1) + 1 out, so *HF* = 1, 2 cm^3s-1 flows through *F* and 1 cm^3s^{-1} leaves along *FE*, leaving 1 cm^3s^{-1} to flow along *FC*.

	(vi)	Applying the information from (ii), (iii) and (iv), together with the new information that the flow in *GF* = 1 cm^3s^{-1} and the total flow is 9 cm^3s^{-1}, ties down the flow in every arc. *AB* = 3, *AC* = 2 and *AD* = 4. The arcs *AB*, *DG*, *DC, FC, FE* and *HE* are all saturated.
	(vii)	The flow can then be increased by sending another 2 cm^3s^{-1} along *AC*. With this flow, *AD* is not saturated, although no more can flow through *AD* because *DG* and *DC* are both saturated. The candidates who understood the difference between capacity and flow were able to state that arcs *AB*, *AC*, *DC* and *DG* are all saturated (along with *FC*, *FE* and *HE*).This gives a saturated cut through *AB*, *AC*, *DC* and *DG*, with the source (*A*) one side and both sinks (*C* and *E*) the other side. This cut is {*A*, *D*}, {*B*, *C*, *E*, *F*, *G*, *H*}. A second possibility is {*A*, *D*, *F*, *G*, *H*},{*B*, *C*, *E*}. All other cuts through saturated arcs have *C* on the same side as *A.* There was the usual confusion between capacity and flow, with candidates offering the cut {*A*}, {*B*, *C*, *D*, *E*, *F*, *G*, *H*}, but *AD* is not saturated; or {*A*, *B*}, {*C*, *D*, *E*, *F*, *G*, *H*}, but *BE*, *BC* and *AD* are not saturated.

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