OCR
Oxford Cambridge and RSA

# Friday 24 June 2016 – Morning

## A2 GCE MATHEMATICS (MEI)

**4754/01A** Applications of Advanced Mathematics (C4) Paper A

**QUESTION PAPER**



Candidates answer on the Printed Answer Book.

**OCR supplied materials:**
- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**
- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

**Section A** (36 marks)

**1** Express $\cos\theta - 3\sin\theta$ in the form $R\cos(\theta+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Hence show that the equation $\cos\theta - 3\sin\theta = 4$ has no solution. **[6]**

**2** Given that $\left(1+\frac{x}{p}\right)^q = 1 - x + \frac{3}{4}x^2 + \ldots$, find $p$ and $q$, and state the set of values of $x$ for which the expansion is valid. **[7]**

**3** Fig. 3 shows the curve $y = x^4$ and the line $y = 4$.

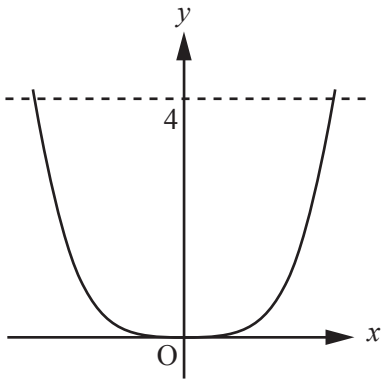


**Fig. 3**

The finite region enclosed by the curve and the line is rotated through 180° about the $y$-axis. Find the exact volume of revolution generated. **[4]**

**4** Solve the equation $2\sin 2\theta = 1 + \cos 2\theta$ for $0° \leqslant \theta \leqslant 180°$. **[5]**

**5** In Fig. 5, triangles ABC, ACD and ADE are all right-angled, and angles BAC, CAD and DAE are all $\theta$.

AB = $x$ and AE = $2x$.

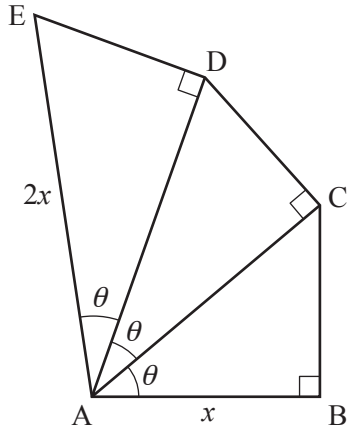


**Fig. 5**

**(i)** Show that $\sec^3\theta = 2$. **[3]**

**(ii)** Hence show the ratio of lengths ED to CB is $2^{\frac{2}{3}} : 1$. **[4]**

**6** P is a general point on the curve with parametric equations $x = 2t, y = \frac{2}{t}$. This is shown in Fig. 6. The tangent at P intersects the $x$- and $y$-axes at the points Q and R respectively.

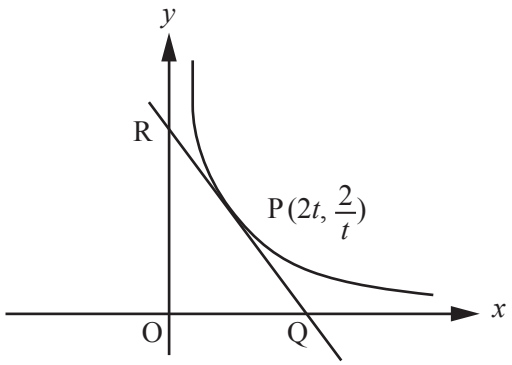


**Fig. 6**

Show that the area of the triangle OQR, where O is the origin, is independent of $t$. **[7]**

**Section B** (36 marks)

**7** Fig. 7 shows a cuboid OABCDEFG with coordinates as shown. The point P has coordinates (4, 2, 0).

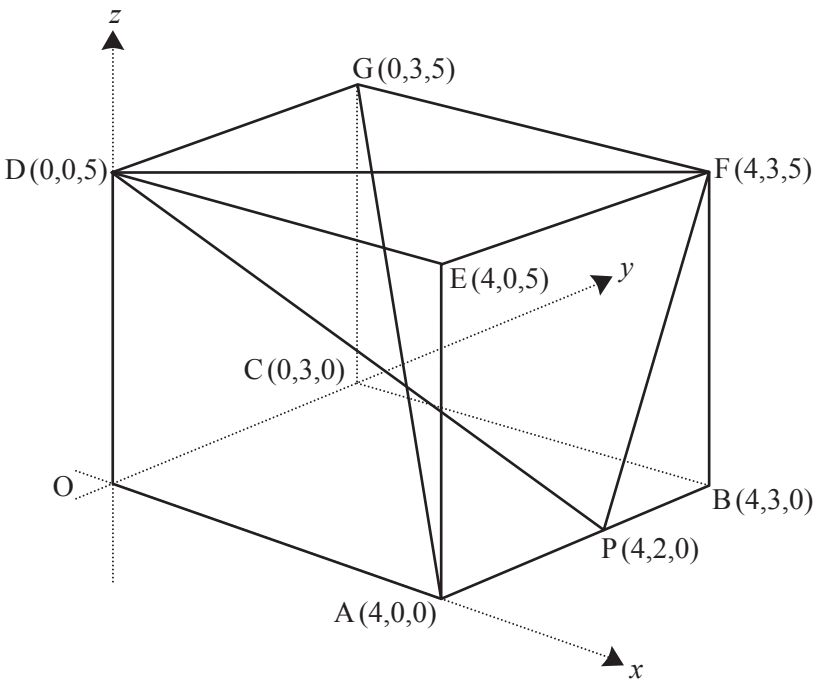


**Fig. 7**

**(i)** Find the length of the diagonal AG. **[2]**

**(ii)** Show that the vector $\mathbf{n} = 15\mathbf{i} - 20\mathbf{j} + 4\mathbf{k}$ is normal to the plane DPF. Hence find the cartesian equation of this plane. **[6]**

The diagonal AG intersects the plane DPF at Q.

**(iii)** Write down a vector equation of the line AG. Hence find the coordinates of the point Q, and the ratio AQ : QG. **[6]**

**(iv)** Find the acute angle between the line AG and the plane DPF. **[4]**

**8** **(i)** Show that $\frac{1}{2+x}+\frac{1}{2-x}=\frac{4}{(2+x)(2-x)}$. **[1]**

In a chemical reaction, the time $t$ minutes taken for a mass $x$ mg of a substance to be produced is modelled by the equation

$$t=\ln\left(\frac{2+x}{2-x}\right).$$

**(ii)** Show that when $t = 0$, $x = 0$. **[2]**

**(iii)** Show that the rate of change of $x$ is proportional to the product of $(2 + x)$ and $(2 – x)$, and find the constant of proportionality. **[4]**

**(iv)** Show that $x=\frac{2(1-\mathrm{e}^{-t})}{1+\mathrm{e}^{-t}}$.

Hence determine the long-term mass of the substance predicted by this model. **[4]**

In another chemical reaction, the mass $x$ mg at time $t$ minutes is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t}=k(2+x)(2-x)\mathrm{e}^{-t},$$

where $k$ is a positive constant, and $x = 0$ when $t = 0$.

**(v)** Show by integration that, for this reaction, $\ln\left(\frac{2+x}{2-x}\right)=4k(1-\mathrm{e}^{-t})$. **[5]**

**(vi)** Given that the long-term mass of this substance is 1.85 mg, find the value of $k$. **[2]**

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