# Section Check In – Pure Mathematics: Algebra

## Questions

1. Solve the inequality .

2. Simplify .

3.\* Express  in partial fractions.

4.\* Simplify .

5. (a) Express each of  and  in completed square form.

 (b) Sketch the curves  and  on the same axes, and show that the shortest distance between the two curves is .

6. (a) Show that .

 (b) Two numbers have mean 7 and product 36. Find the numbers.

7. Show that the straight line  meets the curve  in two distinct points for all values of the constant .

8. The height in metres and weight in kilograms for a random sample of three women is shown in the table below.

|  |  |
| --- | --- |
| **Height (m)** | **Weight (kg)** |
| 1.47 | 54.5 |
| 1.48 | 62.6 |
| 1.55 | 90.4 |

 A model for human weight is that it is proportional to the cube of height. Use the data in the table to determine whether the model appears to be suitable.

9. An object is projected vertically upwards from ground level and its height, metres, after  seconds is given by the formula

.

 Show that the object is above a height of metres for approximately seconds.

10. Solve the inequality .

**Extension**

The table below shows the number of days of sickness absence for employees in a business in one year. The frequency row shows the number of employees with that number of days of sickness absence. No-one had over 20 days’ absence.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Mean days (*x*)** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| **Frequency (*y*)** | 338 | 10 384 | 2138 | 1011 | 553 | 332 | 248 | 184 | 141 | 103 |

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Mean days (*x*)** | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **Frequency (*y*)** | 95 | 75 | 60 | 53 | 48 | 36 | 35 | 33 | 27 | 24 | 22 |

(a) A model of the form *y* is inversely proportion to  is proposed. Find a suitable integer value of *k*.

(b) Give one way in which the model could be improved.

(c) Models of this form are called power functions. Find some other applications of power functions.

## Worked solutions

1.  so .

  BC

2. Simplifying, 

3. Let 

 Multiplying by  gives 

 Substituting  gives  and therefore 

 Substituting  gives  and therefore 

 Expression is 

4. Expression is 

 Factorising the quadratic expressions, 

 Cancelling factors common to numerator and denominator, expression is 

5. (a) 

 

 (b)  has minimum at 

  has maximum at 

*y*

1

27

*x*









 Shortest distance between curves is the distance between their stationary points.

 Shortest distance is 

6. (a) .

 (b) Suppose the numbers are  and 

 Product is 

 

 Numbers are  and .

7. Substituting  in the equation of the curve, 

 Expanding, 

 Simplifying, 

 Discriminant, 

 For all values of ,  and therefore 

Since the discriminant is positive, the quadratic equation has two distinct roots and so the line meets the curve in two distinct points.

8. If weight (*w*) is proportional to height cubed, where *h* is the height then  where *k* is a constant.

 .

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  (to 3 d.p.) |
| 1.47 | 54.5 | 3.176523 | 17.157 |
| 1.48 | 62.6 | 3.241792 | 19.310 |
| 1.55 | 90.4 | 3.723875 | 24.276 |

 The values in the last column are not constant so the model does not appear to be suitable.

9. Times when object is at a height of metres found by substituting 

 Equation is  which simplifies to 

 Using quadratic formula,  or 

Object at height of metres after seconds (going up) and after seconds (going down)

Object above this height for seconds, i.e. for seconds (to 3 significant figures)

10.  is positive so both sides of the inequality can be multiplied by it.

 

 

 

 The sketch shows .



 *y* is negative when the curve is below the *x*-axis.

 

**Extension**

(a) If *y* is inversely proportion to then  should be constant. Working outfor different values of *k* using a spreadsheet gives the following results. The values are nearest to constant for  so 2 is a suitable value for *k*.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *x* | *y* |  |  |  |  |
| 0 | 338 | 0 | 0 | 0 | 0 |
| 1 | 10384 | 10384 | 10384 | 10384 | 10384 |
| 2 | 2138 | 4276 | 8552 | 17104 | 34208 |
| 3 | 1011 | 3033 | 9099 | 27297 | 81891 |
| 4 | 553 | 2212 | 8848 | 35392 | 141568 |
| 5 | 332 | 1660 | 8300 | 41500 | 207500 |
| 6 | 248 | 1488 | 8928 | 53568 | 321408 |
| 7 | 184 | 1288 | 9016 | 63112 | 441784 |
| 8 | 141 | 1128 | 9024 | 72192 | 577536 |
| 9 | 103 | 927 | 8343 | 75087 | 675783 |
| 10 | 95 | 950 | 9500 | 95000 | 950000 |
| 11 | 75 | 825 | 9075 | 99825 | 1098075 |
| 12 | 60 | 720 | 8640 | 103680 | 1244160 |
| 13 | 53 | 689 | 8957 | 116441 | 1513733 |
| 14 | 48 | 672 | 9408 | 131712 | 1843968 |
| 15 | 36 | 540 | 8100 | 121500 | 1822500 |
| 16 | 35 | 560 | 8960 | 143360 | 2293760 |
| 17 | 33 | 561 | 9537 | 162129 | 2756193 |
| 18 | 27 | 486 | 8748 | 157464 | 2834352 |
| 19 | 24 | 456 | 8664 | 164616 | 3127704 |
| 20 | 22 | 440 | 8800 | 176000 | 3520000 |

(b) Possible answers include the following.

* Try non-integer powers.
* Use a different formula for low values of *x*.

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